

考試科目 Course	數理統計	開課系級 Dept. & Class	研究所	日期 Date, Period	2019 年 3 月 4 日 上午 9:00~12:00	試題編號 Course No.
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本試卷共有 6 個題目，
 博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Let X and Y be two random variables.

(a) Define

$$X \wedge Y = \min(X, Y) \quad \text{and} \quad X \vee Y = \max(X, Y)$$

Show that $E(X \vee Y) = E(X) + E(Y) - E(X \wedge Y)$

(b) If X is a continuous and nonnegative random variable, then show that $E(X) = \int_0^{\infty} 1 - F_X(x) dx$, where $F_X(x)$ is the cdf of X .

2. Suppose X has a binomial (n, p) distribution and let Y have a negative binomial (r, p) distribution.

(a) Show that $F_X(r-1) = 1 - F_Y(n-r)$, where F_X and F_Y are the cdfs of X and Y , respectively.

(b) Show that a negative binomial family with r known, $0 < p < 1$, is an exponential family.

3. Let X and Y be independent random variables with the same geometric distribution. Define

$$U = \min(X, Y) \quad \text{and} \quad V = X - Y$$

Show that U and V are independent.

本考試： 不需使用簡易計算機， 使用簡易計算機

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命題老師：
(Teacher)

試題隨卷繳交

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Remarks : For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.

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4. Show that the sequence of random variables, X_1, X_2, \dots , converges in probability to a constant μ if and only if the sequence also converges in distribution to μ .

5. Let X_1, X_2, \dots, X_n be a random sample from the distribution with the following pdf:

$$f(x|\theta) = e^{-(x-\theta)}, \quad \theta < x < \infty, \quad -\infty < \theta < \infty$$

(a) Find a minimal sufficient statistic for θ .

(b) Find the Uniformly Minimal Variance Unbiased Estimator (UMVUE) for θ .

6. Suppose that we have two independent random samples:

X_1, X_2, \dots, X_n are exponential (θ),

and

Y_1, Y_2, \dots, Y_m are exponential (μ).

(a) Find the Likelihood Ratio Test (LRT) of

$$H_0: \theta = \mu \quad \text{versus} \quad H_1: \theta \neq \mu$$

(b) Show that the test in (a) can be based on the statistic

$$T = \frac{\sum_{i=1}^n X_i}{\left(\sum_{i=1}^n X_i + \sum_{i=1}^m Y_i\right)}$$

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