國立政治大學應用數學系九十五學年度第一學期研究生學科考試試題

科目：作業研究

1. A company reviews the state of one of its important products annually and decides whether it is successful (state 1) or unsuccessful (state 2). The company must decide whether or not to advertise the product to further promote sales. The following matrices, P^1 and P^2, provide the transition probabilities with and without advertisement during any year. The associated return are given by the matrices R^1 and R^2. Find the optimal decision over the next 3 years.

$$P^1 = \begin{pmatrix} .9 & .1 \\ .6 & .4 \end{pmatrix},\ R^1 = \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix},\ P^2 = \begin{pmatrix} .7 & .3 \\ .2 & .8 \end{pmatrix},\ R^2 = \begin{pmatrix} 4 & 1 \\ 2 & -1 \end{pmatrix}.$$

2. *Dual Simplex with Artificial Constraints.* Consider the following problem:

$$\begin{aligned} \max \quad & z = 2x_1 - x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 - 5x_3 \geq 4 \\ & -x_1 + 9x_2 - x_3 \geq 3 \\ & 4x_1 + 6x_2 + 3x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

The starting basic solution consisting of surplus x_4 and x_5 and slack x_6 is infeasible because $x_4 = -4$ and $x_5 = -3$. However, the dual simplex is not applicable directly because x_1 and x_3 do not satisfying the maximization optimality condition. Show that by augmenting the artificial constraint $x_1 + x_3 \leq M$ (where M is sufficiently large not to eliminate any feasible points in the original solution space), and then using the new constraint as a pivot row, the selection of x_1 as the entering variable (because it has the most negative objective coefficient) will render an all-optimal objective row. Next, carry out the regular dual simplex method on the modified problem.

3. A company has a contract to provide workers over the next 4 months according the following schedules:

Month	Jan.	Feb.	Mar.	Apr.
No. of workers	100	120	80	170

Because of change in demand, it may be economical to retain more workers than needed in a given month. The cost of recruiting and maintaining a worker is a function of their employment period as the following table shows:

Employment period (months)	Jan.	Feb.	Mar.	Apr.
Cost per worker ($)	100	130	180	220

Let x_{ij} = number of workers hired at the start of month i and terminated at the start of month j. So, x_{12} gives the number of worker hired in Jan. for one month only. x_{45} defines hiring in April for April. Develop the linear program and the associated minimum-cost flow network for this problem.

4. Consider the following problem:

$$\begin{array}{ll} \max & z = 2x_1 - x_2 + x_3 \\ \text{s.t.} & 3x_1 - 2x_2 + 2x_3 \leq 15 \\ & -x_1 + x_2 + x_3 \leq 3 \\ & x_1 - x_2 + x_3 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

If we let x_4, x_5, and x_6 be the slack variables for the respective constraints, the simplex method yields the following final set of equations:

$$\begin{array}{l} (0)\ z + 2x_3 + x_4 + x_5 = 18 \\ (1)\ x_2 + 5x_3 + x_4 + 3x_5 = 24 \\ (2)\ 2x_3 + x_5 + x_6 = 7 \\ (3)\ x_1 + 4x_3 + x_4 + 2x_5 = 21 \end{array}$$

Now you are conduct sensitivity analysis by independently investigating each of the following changes in the original model.

(a) Change constraint 1 to $2x_1 - x_2 + 4x_3 \leq 12$.

(b) Change the objective function to $z = 5x_1 + x_2 + 3x_3$.

(c) Introduce a new constraint $2x_1 + x_2 + 3x_3 \leq 60$.

5. A cafeteria can seat a maximum of 20 persons. Customers arrive in a Poisson stream at the rate of 10 persons per hour and are served (one at a time) at the rate of 12 per hour.

(a) What is the probability that an arriving customer will not eat in the cafeteria because it is full?

(b) Suppose that three customers (with random arrival times) would like to be seated together. What is the probability that their wish can be fulfilled? (Assume that arrangements can be made to seat them together as long as three seats are available.)

6. Consider the following LP model:

$$\begin{array}{ll} \max & z = 2x_1 + x_2 \\ \text{s.t.} & 3x_1 + x_2 - x_3 = 3 \\ & 4x_1 + 3x_2 - x_4 = 6 \\ & x_1 + 2x_2 + x_5 = 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

Compute the entire simplex tableau associate with the following basic solution and check it for optimality and feasibility.

$$\text{Basic variables} = (x_1, x_2, x_5),\ \text{Inverse} = \begin{pmatrix} 3/5 & -1/5 & 0 \\ -4/5 & 3/5 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$