1. Apply revised simplex method to the following problem by two iterations:

$$
\begin{aligned}
\max \quad & 3x_1 + 4x_2 + x_3 + x_4 \\
\text{s.t.} \quad & 8x_1 + 3x_2 + 4x_3 + x_4 \leq 20 \\
& 2x_1 + 6x_2 + x_3 + 5x_4 \leq 4 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

   At each iteration, identify the dual variables and show which dual constrains are violated.

2. Solve the following linear program by the decomposition method. Show the progress of the lower bound and primal objective. Obtain primal and dual solution.

$$
\begin{aligned}
\min \quad & -2x_1 - x_2 - x_3 + x_4 \\
\text{s.t.} \quad & x_1 + x_3 \leq 2 \\
& x_1 + x_2 + 2x_4 \leq 3 \\
& x_1 \leq 2 \\
& x_1 + 2x_2 \leq 5 \\
& -x_3 + x_4 \leq 2 \\
& 2x_3 + x_4 \leq 6 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

3. Consider the following problem:

$$
\begin{aligned}
\max \quad & z = 2x_1 + 7x_2 - 3x_3 \\
\text{s.t.} \quad & x_1 + 3x_2 + 4x_3 \leq 30 \\
& x_1 + 4x_2 - x_3 \leq 10 \\
& x_j \geq 0, j = 1, 2, 3
\end{aligned}
$$

   (a) By letting x_4 and x_5 be the slack variables for the respective constrains, solve the problem by revised simplex method, and write down the final set of equations.

   Now you are to conduct sensitivity analysis by independently investigating each of the following changes in the original model. For each change, use the sensitivity analysis procedure to revise this set of equations (in tableau form) and convert it to proper form from Gaussian elimination for identifying and evaluating the current basic solution. Then test this solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

   (b) Change the right side to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$.

   (c) Change the objective function to $z = x_1 + 5x_2 - 2x_3$.

   (d) Introduce a new constraint $3x_1 + 2x_2 + 3x_3 \leq 25$.

4. A library is planning the purchase of shelves to stock its collection of books. The library has books that come in various thickness and heights. Let $h_1 < h_2 < \cdots < h_n$ be the possible values of book heights and let l_i be the total thickness of all book with height h_i. A book of height h can be stored in a shelf of any height greater than or equal to h. Note that books cannot be double stacked, i.e., if a small book is placed in a large shelf the space above the book is wasted. Let x_i denote the total length of the shelf space of height h_i that is constructed. The associated cost is 0 if $x_i = 0$; and is $k_i + c_i x_i$ if $x_i > 0$, where k_i is a known fixed charge for construction and c_i is the cost per unit length for shelves of height h_i.

   (a) Show that the problem can be modeled as the shortest-route problem. (Calculate the associated d_{ij}'s and draw the graph for this problem.)

   (b) Solve this problem by using Dijkstra's algorithm for the following data: $(h_1, h_2, h_3, h_4) = (5, 7, 9, 12)$; $(l_1, l_2, l_3, l_4) = (3, 4, 18, 12)$; $(k_1, k_2, k_3, k_4) = (7, 7, 9, 9)$; $(c_1, c_2, c_3, c_4) = (6, 8, 9, 12)$.

5. Suppose that the one-step transition matrix of a Markov chain is as follows:

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0 & 0.6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) For each absorbing state, find the probability of absorption into that state.

   (b) For each transient state, find the mean amount of time spent occupying that state.

   (c) The mean amount of time spent before absorption.

6. For an M/M/1 queuing system, suppose that both λ and μ are doubled.

   (a) How is L changed?

   (b) How is W changed?

   (c) How is the steady-state probability distribution changed?