國立政治大學應用數學系九十三學年度第一學期研究生學科考試試題

科目：作業研究

1. Consider the transportation problem corresponding to the following tableau.

	1	2	3	4	s_i
1	4	3	6	5	20
2	7	10	5	6	30
3	8	9	7	7	50
d_j	15	35	20	30	

c_{ij} matrix

(a) Solve the problem by the transportation algorithm.

(b) Suppose that c_{24} is replaced by 4. Without resolving the problem, find the new optimal solution.

(c) How large should c_{12} be made before some optimality condition of the solution in part (a) is violated.

2. Use the *dual simplex method* to solve the following problem:

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 + 4x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \geq 15 \\ & 2x_1 - x_2 + 3x_3 \geq 20 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. Consider a linear programming in its standard form with $P = \{\boldsymbol{x} \in R^n \mid \boldsymbol{Ax} = \boldsymbol{b},\ \boldsymbol{x} \geq \boldsymbol{0}\}$.

(a) Let $\boldsymbol{d} \in R^n$. Show that a necessary condition for $\boldsymbol{d}$ to be a feasible direction is that $\boldsymbol{Ad} = \boldsymbol{0}$.

(b) Suppose $\boldsymbol{x} = (x_1, \ldots, x_n)^T \in P$ with $x_i > 0$ when $d_i \neq 0$. Show that there exist a scalar $\alpha > 0$ such that $\boldsymbol{x} + \alpha\boldsymbol{d} \geq \boldsymbol{0}$.

4. Show that the following two systems:

$$\begin{aligned} &\text{(I)} \quad \boldsymbol{Ax} = \boldsymbol{b},\ \boldsymbol{x} \geq \boldsymbol{0} \\ &\text{(II)} \quad \boldsymbol{wA} \leq \boldsymbol{0},\ \boldsymbol{wb} > \boldsymbol{0} \end{aligned}$$

either system (I) or (II) is solvable, but not both, where $\boldsymbol{A}$ is an $m \times n$ matrix with full rank; $\boldsymbol{x}$ is an n-dimensional column vector; $\boldsymbol{w}$ is an m-dimensional column vector; and $m \leq n$. (Hint: Let $\boldsymbol{c} = \boldsymbol{0}$ then uses duality theorem.)

5. Suppose that the one-step transition matrix of a Markov chain is as follows:

$$\boldsymbol{P} = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) For each absorbing state, find the probability of absorption into that state.

(b) For each transient state, find the mean amount of time spent occupying that state.

(c) The mean amount of time spent before absorption.

6. An appliance store can place orders for refrigerators at the beginning of each month for immediate delivery. A fixed cost of \$100 is incurred every time an order is placed. The storage cost per refrigerator per month is \$5. The penalty for running out of stock is estimated at \$150 per refrigerator per month. The month demand is given by the following pdf:

Demand x	0	1	2
$p(x)$	0.2	0.5	0.3

The store policy is that the maximum stock level should not exceed two refrigerators in any single month. Determine the following:

(a) The transition probabilities for the different decision alternatives of the problem.

(b) The expected inventory cost per month as a function of the state of the system and the decision alternative.

(c) The optimal ordering policy over the next 3 months.