1. The demand for a perishable item over the next four months is 400, 300, 420, and 380 tons, respectively. The supply capacities for the same months are 500, 600, 200, and 300 tons. The Purchase price per ton varies from month to month and is estimated at \$100, \$140, \$120, and \$150, respectively. Because the item is perishable, a current month's supply must be consumed within 3 months (starting with the current month). The storage cost per ton per month is \$3. The nature of the item does not allow back-ordering.

   a. Define the required decision variables and coefficients. Then write down the mathematical model. (10%)

   b. Solve the problem as the transportation model. (10%)

2. Consider the following LP model:

$$\begin{aligned} \max \quad & z = 5x_1 + 2x_2 + 3x_3 \\ \text{s.t.} \quad & x_1 + 5x_2 + 2x_3 \le b_1 \\ & x_1 - 5x_2 - 6x_3 \le b_2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

   The following optimal tableau corresponds to specific values of b_1 and b_2:

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	a	7	d	e	150
x_1	1	b	2	1	0	30
x_5	0	c	-8	-1	1	10

   Determine the following and write down the calculation procedures to get the value:

   a. The write side value b_1 and b_2 (5%)

   b. The elements a, b, c, d, e. (10%)

   c. The optimal dual solution. (5%)

3. Develop the master and the subproblem using the decomposition technique for the following linear programming. Assume that X is polyhedral and has a special structure. Formally state the decomposition algorithm for this case. (20%)

$$\begin{aligned} \max \quad & \boldsymbol{cx} + \boldsymbol{dy} \\ \text{s.t.} \quad & \boldsymbol{Ax} + \boldsymbol{Dy} \le \boldsymbol{b} \\ & \boldsymbol{x} \in X \end{aligned}$$

4. Consider the problem

$$\begin{aligned} \min \quad & \boldsymbol{cx} \\ \text{s.t.} \quad & \boldsymbol{Ax} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}$$

where $m = n$, $\boldsymbol{c} = \boldsymbol{b}^t$, and $\boldsymbol{A} = \boldsymbol{A}^t$. Show that if there exist an $\boldsymbol{x}_0$ such that $\boldsymbol{Ax}_0 = \boldsymbol{b}$, $\boldsymbol{x}_0 \geq \boldsymbol{0}$, then $\boldsymbol{x}_0$ is an optimal solution. (Hint: Use duality) (20%)

5. At $t = 0$ a customer (the *test customer*) places a request for service and finds all s servers busy and j other customers waiting for service. All customers wait as long as necessary for service, waiting customers are served in order of arrival, and no new requests for service are permitted after $t = 0$. Service times are assumed to be mutually independent, identically distributed, exponential random variables, each with mean duration μ^{-1}.

   a. Find the expected length of time the test customer spends waiting for service in the queue (10%)

   b. Let X be the order of completion of service of the test customer; that is, $X = m$ if the test customer is the mth customer to complete service after $t = 0$. Find $P\{X = m\}$, $m = 1, 2, \ldots, s + j + 1$. (10%)

6. In a simplified Monopoly game there are four places on the board numbered 0 through 3. The matrix shows the transition probabilities between each of them on any one play of the game. Let the places be states.

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

   a. Find the steady state probabilities for the corresponding Markov chain. (10%)

   b. If you receive \$3 every time you reach state 2 and pay \$1 when you reach any of the other states what is your excepted profit per play (in steady state). (5%)

   c. If you start in state 0, what is you long-run excepted return? (5%)