每題20分，任選五題作答

1. Solve the following KLP problem.

$$\begin{aligned} \min \quad & z = x_2 \\ \text{s.t.} \quad & x_1 + x_2 - 2x_3 = 0 \\ & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

   (a) You only need to perform 2 iterations, where $r = 1/\sqrt{6}$ and $\alpha = 2/9$. (5%)

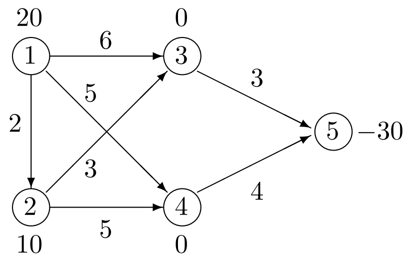
   Consider the preview problem if the objective function is changed to $z = 2x_1 + 5x_2 + x_3$.

   (b) What is the improving direction? (5%)

   (c) What is the steepest descent feasible direction? (5%)

   (d) Consider a point at $(2/9, 4/9, 1/3)$, gives a transformation that transform this point to the center of the simplex. (5%)

2. Consider the minimum cost flow problem shown below, where the f_i values are given by the nodes, the c_{ij} values are given by the arcs, and the u_{ij} values are given to the right by the graph.



Arc capacities
$1 \to 3 : 10$
$2 \to 3 : 25$
others : ∞

   (a) Write down the mathematical model and the associate dual model. (10%)

   (b) Obtain an initial BF solution by solving the feasible spanning tree with basic arcs $1 \to 2$, $3 \to 5$, $4 \to 5$, and $1 \to 3$, where one of the nonbasic arcs $(3 \to 2)$ is a reverse arc. (10%)

3. Consider the problem on 2. Use the optimality test to verify that this initial BF solution is optimal and that there are multiple optimal solutions. Apply one iteration of the *network simplex* method to find the other optimal solution, and then use these results to identify the other optimal solution that are not BF solution. (20%)

4. Consider the following primal problem

$$\begin{aligned} \min \quad & \mathbf{cx} \\ \text{s.t.} \quad & \mathbf{Ax} \geq \mathbf{b} \\ & \mathbf{x} \in X \end{aligned}$$

where X is a polyhedral set. (Often the set X consists of constraints that are easy to handle.) Associated with the foregoing primal problem is the following *Lagrangian dual* problem.

$$\begin{aligned} \max \quad & f(\mathbf{w}) \\ \text{s.t.} \quad & \mathbf{w} \geq 0 \end{aligned}$$

where $f(\mathbf{w}) = \mathbf{wb} + \min_{\mathbf{x} \in X} (\mathbf{c} - \mathbf{wA})\mathbf{x}$.

(a) Show that if $\mathbf{x}_0$ is feasible to the primal problem, that is $\mathbf{Ax}_0 \geq \mathbf{b}$ and $\mathbf{x}_0 \in X$, and $\mathbf{w}_0$ is feasible to the Lagrangian dual problem, that is $\mathbf{w}_0 \geq 0$, then $\mathbf{cx}_0 \geq f(\mathbf{w}_0)$. (10%)

(b) Suppose that X is nonempty and bounded and that the primal problem processes a finite optimal solution. Show that $\min\{\, \mathbf{cx} \mid \mathbf{Ax} \geq \mathbf{b}, \mathbf{x} \in X \,\} = \max\{\, f(\mathbf{w}) \mid \mathbf{w} \geq 0 \,\}$. (10%)

5. Consider the following problem:

$$\begin{aligned} \min \quad & x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \geq 6 \\ & -x_1 + x_2 \leq 2 \\ & x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Let $X = \{\, \mathbf{x} \mid -x_1 + x_2 \leq 2, x_1 + x_2 \leq 8, x_1, x_2 \geq 0 \,\}$

(a) Formulate the Lagrangian dual problem. (10%)

(b) Show that $f(\mathbf{w}) = 6w + \min\{\, 0, 4 - 2w, 13 - 14w, 8 - 24w \,\}$. (10%)

6. Use the decomposition technique to solve the following problem. (20%)

$$\begin{aligned} \max \quad & x_1 + 5x_2 + 4x_3 + 3x_4 \\ \text{s.t.} \quad & 2x_1 + 4x_2 + 5x_3 + 2x_4 \leq 7 \\ & 2x_1 + 3x_2 \leq 6 \\ & 5x_1 + x_2 \leq 5 \\ & 3x_3 + 4x_4 \geq 12 \\ & x_3 \leq 4, x_4 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$