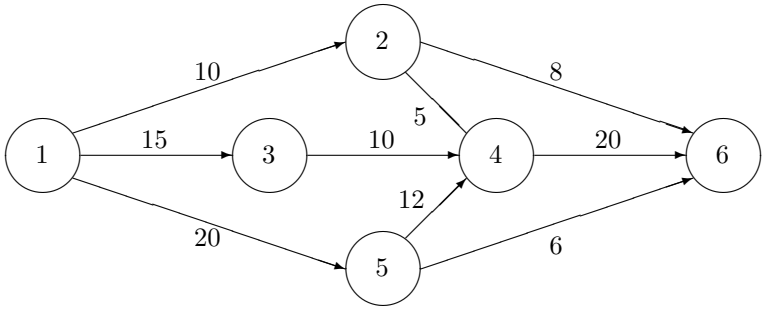
每題20分，任選五題作答

1. Consider the following network where 1 is the source node, 6 is the sink node, and the numbers along the arcs denote the capacities of flows:



   (a) Illustrate a *cut* separating the source and sink and give the *capacity* of your cut. (5%)

   (b) Find the maximum flow from the source to the sink using augmenting path algorithm. Identify the associated minimum cut of your solution, and verify the max-flow min-cut theorem. (15%)

2. (a) For each of the following LPs (I and II) give your recommendation on what is probably the more efficient way for obtaining an optimal solution: Solving the primal by the (*Dual*) *Simplex Method* or solving the dual by the (*Dual*) *Simplex Method*. Explain why. (10%)

   I. $\max \ 7x_1 + 3x_2 - 10x_3$
   $\text{s.t.} \ \ 2x_1 - x_2 + 3x_3 \le 40$
   $3x_1 + 2x_2 - x_3 = 35$
   $2x_1 + x_2 + 5x_3 \le 20$
   $x_1 + x_2 - x_3 \le 60$
   $x_1 + x_2 + 2x_3 \ge 10$
   $x_1 \ge 0, \ x_2 \ge 0, \ x_3 \le 0$

   II. $\max \ x_1 + 4x_2 + x_3 + x_4 + 2x_5$
   $\text{s.t.} \ \ x_1 + 4x_2 + 5x_3 + 3x_4 + x_5 \ge 9$
   $x_1 + 7x_2 + 4x_3 + 6x_4 + 4x_5 \le 22$
   $x_j \ge 0, \ \forall j = 1, \ldots, 5$

   Do not solve any LP.

   (b) Write down the dual problems of I and II. (10%)

3. Consider the following problem:

$$\begin{aligned} \max \quad & z = 2x_1 + 7x_2 - 3x_3 \\ \text{s.t.} \quad & x_1 + 3x_2 + 4x_3 \le 30 \\ & x_1 + 4x_2 - x_3 \le 10 \\ & x_j \ge 0, \ j = 1, 2, 3 \end{aligned}$$

(a) By letting x_4 and x_5 be the slack variables for the respective constraints, solve the problem by revised simplex method, and write down the final set of equations. (5%)

Now you are to conduct sensitivity analysis by independently investigating each of the following changes in the original model. For each change, use the sensitivity analysis procedure to revise this set of equations (in tableau form) and convert it to proper form from Gaussian elimination for identifying and evaluating the current basic solution. Then test this solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

(b) Change the right side to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$. (5%)

(c) Change the objective function to $z = x_1 + 5x_2 - 2x_3$. (5%)

(d) Introduce a new constraint $3x_1 + 2x_2 + 3x_3 \leq 25$. (5%)

4. A library is planning the purchase of shelves to stock its collection of books. The library has books that come in various thickness and heights. Let $h_1 < h_2 < \cdots < h_n$ be the possible values of book heights and let l_i be the total thickness of all book with height h_i. A book of height h can be stored in a shelf of any height greater than or equal to h. Note that books cannot be double stacked, i.e. if a small book is placed in a large shelf the space above the book is wasted. Let x_i denote the total length of the shelf space of height h_i that is constructed. The associated cost is 0 if $x_i = 0$; and is $k_i + c_i x_i$ if $x_i > 0$, where k_i is a known fixed charge for construction and c_i is the cost per unit length for shelves of height h_i.

   (a) Show that the problem can be modeled as the shortest-route problem. (Calculate the associated d_{ij}'s and draw the graph for this problem.) (10%)

   (b) Solve this problem by using Dijkstra's algorithm for the following data: $(h_1, h_2, h_3, h_4) = (5, 7, 9, 12)$; $(l_1, l_2, l_3, l_4) = (3, 4, 18, 12)$; $(k_1, k_2, k_3, k_4) = (7, 7, 9, 9)$; $(c_1, c_2, c_3, c_4) = (6, 8, 9, 12)$; (10%)

5. Suppose that the one-step transition matrix of a Markov chain is as follows:

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.3 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0 & 0.6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) For each absorbing state, find the probability of absorption into that state. (10%)

(b) For each transient state, find the mean amount of time spent occupying that state. (5%)

(c) The mean amount of time spent before absorption. (5%)

6. A small town has a grain elevator. During the harvest season, trucks loads with wheat from the fields arrive at the elevator, where they must quickly deposit their loads and return to the field for another. The check-in process involves weighting the truck, drawing samples for moisture and contamination tests, and a few other details, before the load can be dumped through a grate. Suppose the average interarrival time for trucks is 6 minutes and the average service time is 5 minutes. Using the M/M/1 model, the average time at the elevator (W) is considered by the farmers to be intolerable. At a meeting of the farmer's cooperative, three suggestions have been proposed: (1) By adding sideboards to the trucks the average interarrival time could lengthened to 10 minutes. (2) Adding of another complete check-in station would double the service capacity. Arriving trucks would join a single line, and the truck at the front of the line would move to the first available of the two check-in point. (3) The grain elevator facility is duplicated on the other side of town. This proposal would split the arrivals into equal halves, since each farmer would attend the nearer of the two elevators.

   Compare the existing system and the three proposals by the factors of L, L_q, W, and W_q then make your suggestion.