每題20分，任選五題作答

1. Glassco manufactures glasses: wine, beer, champagne, and whiskey. Each type of glass requires time in the molding shop and in the packaging shop, and a certain amount of glass. At present, 600 minutes of molding time, 400 minute of packaging time, and 500 oz of glass are available. After consider the resource required constraints and the maximizing of profit, the problem can be formulated as follows:

$$
\begin{array}{lll}
\max & z = 6x_1 + 10x_2 + 9x_3 + 20x_4 & \\
\text{s.t.} & 4x_1 + 9x_2 + 7x_3 + 10x_4 \le 600 & \text{(Molding constraint)} \\
 & x_1 + x_2 + 3x_3 + 40x_4 \le 400 & \text{(Packaging constraint)} \\
 & 3x_1 + 4x_2 + 2x_3 + x_4 \le 500 & \text{(Glass constraint)} \\
 & x_1, x_2, x_3, x_4 \ge 0 &
\end{array}
$$

It can be shown that the optimal solution to this LP is $z = \frac{2800}{3}, x_1 = \frac{400}{3}, x_4 = \frac{20}{3}, x_2 = x_3 = 0, s_1 = s_2 = 0, s_3 = \frac{280}{3}$.

(a) Estimate an upper bound for this problem? State your procedure. (5%)

(b) Find the dual of the Glassco problem. (5%)

(c) Using the giving optimal primal solution and the Theorem of Complementary Slackness, find the optimal solution to the dual of the Glassco problem. (5%)

(d) If the manager of the Glassco has more budgets. What resource he shall be purchased to increase the profit? Why? (5%)

2. I have just purchased (at time 0) a new car for \$12,000. The cost of maintaining a car during a year depends on the age of the car at the beginning of the year, as given in following table. To avoid the high maintenance costs associated with an older car, I may trade in my car and purchase a new car. The price I receive on a trade-in depends on the age of the car at the time of trade-in (see table). To simplify the computations, we assume that at any time, it costs \$12,000 to purchase a new car. My goal is to minimize the net cost (purchasing costs + maintenance costs money received in trade-ins) incurred during the next five years.

Age of Car (years)	Annual maintenance cost	Trade-in Cost
0	\$2000	- - -
1	\$4000	\$7000
2	\$5000	\$6000
3	\$9000	\$2000
4	\$12000	\$1000
5	- - -	\$0

(a) Formulate this problem as a shortest path problem. (10%)

(b) Use Dijkstra's algorithm to find the solution of the problem. (10%)

3. Use the Wagner-Whitin and Silver-Meat methods to find production schedules for the following dynamic lot size problem: $K = \$50, h = \$0.4, d_1 = 10, d_2 = 60, d_3 = 20, d_4 = 140, d_5 = 90$. (20%)

4. My home uses two light bulbs. On average, a light bulb lasts for 22 days (exponentially distributed). When a light bulb burns out, it takes me an average of 2 days (exponentially distributed) before I replaced the bulb.

   (a) Formulate a three-state birth-and-death model of this problem. (10%)

   (b) Determine the fraction of the time that both light bulbs are working. (5%)

   (c) Determine the fraction of the time that no light bulbs are working. (5%)

5. For an $M/M/1$ queuing system, suppose that both λ and μ are doubled.

   (a) How is L changed? (5%)

   (b) How is W changed? (5%)

   (c) How is the steady-state probability distribution changed? (10%)

6. Use the Dantzig-Wolfe decomposition method to solve the following LP. (20%)

$$
\begin{aligned}
\min\quad & 2x_1 - x_2 + x_3 - x_4 \\
\text{s.t.}\quad & x_1 + 2x_2 \leq 4 \\
& x_1 - x_2 \leq 1 \\
& x_3 - 3x_4 \leq 7 \\
& 2x_3 + x_4 \leq 10 \\
& x_1 + 3x_2 - x_3 - 2x_4 \leq 10 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$