1. Apply the primal-dual method to the following problem by two iterations.

$$\begin{aligned} \min \quad & x_1 + 2x_3 - x_4 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \leq 6 \\ & 2x_1 - x_2 + 3x_3 - 2x_4 \geq 5 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

2. Apply revised simplex method to the following problem by two iterations:

$$\begin{aligned} \max \quad & 3x_1 + 4x_2 + x_3 + x_4 \\ \text{s.t.} \quad & 8x_1 + 3x_2 + 4x_3 + x_4 \leq 20 \\ & 2x_1 + 6x_2 + x_3 + 5x_4 \leq 4 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

   At each iteration, identify the dual variables and show which dual constraints are violated.

3. Solve the following linear program by the decomposition method. Show the progress of the lower bound and primal objective. Obtain primal and dual solution.

$$\begin{aligned} \min \quad & -2x_1 - x_2 - x_3 + x_4 \\ \text{s.t.} \quad & x_1 + x_3 \leq 2 \\ & x_1 + x_2 + 2x_4 \leq 3 \\ & x_1 \leq 2 \\ & x_1 + 2x_2 \leq 5 \\ & -x_3 + x_4 \leq 2 \\ & 2x_3 + x_4 \leq 6 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

4. Suppose that an optimal solution to the problem

$$\begin{aligned} \min \quad & \boldsymbol{cx} \\ \text{s.t.} \quad & \boldsymbol{Ax} \geq \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}$$

   exists. Prove the following theories.

   (a) A variable is zero for all primal optimal solution if and only if its complementary dual variable is positive for some dual optimal solution.

   (b) A variable is unbounded in the primal feasible set if and only if its complementary dual variable is bounded in the dual feasible set.

5. The demand for a company's product over the next 12 month is given as d_i, $i = 1, \ldots, 12$. There is a fixed charge of f_i to make a production run. The cost of holding the product in inventory for 1 month is h for each unit remaining in inventory at the end of the month. There is no lead time for production, initial inventory is zero, and no inventory cost is charged for items held less than 1 month. It can be show that it is never optimal to produce when product remains in inventory, and that it is never optimal the produce more than enough to cover an integer number of future months.

   (a) Define the following variable:

   $$x_{ij} = \begin{cases} 1, & \text{if the product in month } i \text{ and covers the demand through month } j, \\ 0, & \text{otherwise,} \end{cases}$$

   $i = 1, \ldots, 12$, $j = 1, \ldots, 12 - i + 1$. Formulate the minimum cost production plan as a set partitioning problem.

   (b) Define the following variables:

   $$\begin{aligned} x_j &= \text{amount of product in month } j \\ y_j &= 1 \text{ if there is nonzero production in month } j\text{; 0 otherwise} \\ z_j &= \text{inventory remaining at the end of month } j \end{aligned}$$

   Give an MILP formulation for the problem.

6. A military maintenance depot overhauls tanks. There is room for three tanks in the facility and one tank in an overflow area outside. At most four tanks can be at the depot at one time. Every morning a tank arrives for an overhaul. If the depot is full, however, it is turned away, so no arrivals occur under these circumstances. When the depot is full, the entire overhaul schedule is delayed 1 day. On any given day, the following probabilities govern the completion of overhauls.

Number of tanks completed	0	1	2	3
Probability	0.2	0.4	0.3	0.1

   These values are independent of the number of tanks in the depot, but obviously no more tanks than are waiting at the start of the day can be completed.

   (a) Develop a Markov chain model for this situation. Begin by defining that state to be the number of tanks in the depot at the start of each day (after the scheduled arrival). Draw the network diagram and write the state-transition matrix.

   (b) Do the same when the state is defined as the number of tanks in the depot at the end of each day.