1. Apply revised simplex method to the following problem by two iterations:

$$\begin{aligned} \max \quad & 2x_1 + x_2 + x_3 + x_4 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + 4x_3 + x_4 \le 16 \\ & 2x_1 + x_2 + x_3 + 2x_4 \le 8 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

   At each iteration, identify the dual variables and show which dual constraints are violated.

2. Solve the following linear program by the decomposition method. Show the progress of the lower bound and primal objective. Obtain primal and dual solution.

$$\begin{aligned} \min \quad & -2x_1 - x_2 - x_3 + x_4 \\ \text{s.t.} \quad & x_1 + x_2 + 2x_4 \le 4 \\ & x_1 \le 2 \\ & x_1 + 2x_2 \le 8 \\ & -x_3 + x_4 \le 2 \\ & 2x_3 + x_4 \le 6 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

3. Show by duality that if the problem

$$\begin{aligned} \min \quad & \boldsymbol{cx} \\ \text{s.t.} \quad & \boldsymbol{Ax} = \boldsymbol{b} \\ & \boldsymbol{x} \ge \boldsymbol{0} \end{aligned}$$

   has a finite optimal solution, then the new problem

$$\begin{aligned} \min \quad & \boldsymbol{cx} \\ \text{s.t.} \quad & \boldsymbol{Ax} = \boldsymbol{b}' \\ & \boldsymbol{x} \ge \boldsymbol{0} \end{aligned}$$

   can not be unbounded, no matter what value the vector $\boldsymbol{b}'$ might take.

4. (a) Transform the following LP to Karmarka's format.

$$\begin{aligned} \max \quad & z = y_1 + 2y_2 \\ \text{s.t.} \quad & y_1 - y_2 \le 2 \\ & 2y_1 + y_2 \le 4 \\ & y_1, y_2 \ge 0 \end{aligned}$$

   (b) Point $\boldsymbol{x} = (6/10, 3/10, 1/10)$ is not in the center of the unit simplex of the 3-dimension space, give a transformation that the point will be at the center of the transformed space.

5. A company sells an item whose demand over the next 4 months is 100, 140, 210, and 180 units, respectively. The company can stock just enough supply to meet each month's demand, or it can overstock to meet the demand for two or more successive and consecutive months. In the latter case, a holding cost of \$1.2 is charged per overstocked unit per month. The company estimates the unit purchase prices for the next 4 months to be \$13, \$15, \$10, and \$12, respectively. A setup cost of \$200 is incurred each time a purchase order is placed. The company wants to develop a purchasing plan that will minimize the total costs of ordering, purchasing, and holding the item in stock.

   (a) Define the following variables:

   $$\begin{aligned} x_j &= \text{amount of purchase in month } j \\ y_j &= 1 \text{ if there is nonzero purchase in month } j\text{; 0 otherwise} \\ z_j &= \text{inventory remaining at the end of month } j \end{aligned}$$

   Give an MILP formulation for the problem.

   (b) Formulate the problem as a shortest-route model and solve it.

6. Consider the birth-and-death process with all $\lambda_n = 2$ $(n = 0, 1, \ldots)$, $\mu_1 = 2$, and $\mu_n = 4$ for $n = 2, 3, \ldots$.

   (a) Display the rate diagram.

   (b) Calculate P_0 and P_1. Then give the general expression for P_n in term of P_0 for $n = 2, 3, \ldots$.