國立政治大學應用數學系九十學年度第二學期研究生學科考試試題

科目：數理統計

本卷共有6個題目，請選5題作答，每題20分。

1. Let X_1, X_2, X_3 be independent with density $f(x_i) = \exp(-x_i)$, $x_i > 0$. Let $U_1 = X_1 + X_2 + X_3$, $U_2 = (X_1 + X_2)/(X_1 + X_2 + X_3)$, and $U_3 = X_1/(X_1 + X_2 + X_3)$. Find the joint density of $\boldsymbol{U} = (U_1, U_2, U_3)$.

2. Suppose that a rat is in a maze with two possible directions. If it chooses left, it wanders around the maze for four minutes and comes back to where it started. If it chooses right, then with probability $1/4$ it will depart the maze in five minutes, and with probability $3/4$ it will come back to where it started after three minutes. We assume that at the beginning and whenever it returns to the start, it chooses to go right with probability $1/5$.

   (a) Find the expected time until the rat escapes the maze.

   (b) Show that the probability that the rat stays in the maze forever is zero.

3. Let $X_1, \ldots, X_n$ be independent, and let $X_i \sim N(\mu, \sigma^2)$, $n > 1$.

   (a) Find the distribution of the sample mean $\overline{X}$.

   (b) Find the distribution of the sample variance S^2.

   (c) Show that $\overline{X}$ and S^2 are independent.

4. Let $X_1, \ldots, X_n$ be independent, and let $X_i \sim N(\mu, \sigma^2)$, $n > 1$. Prove or disprove:

   (a) $\overline{X}$ is the best unbiased estimator of μ.

   (b) S^2 is an efficient estimator of σ^2.

   (c) $\overline{X}S^2$ is an efficient estimator of $\mu\sigma^2$. (Hint: $Var(\overline{X}S^2) = \frac{2\sigma^4}{n(n-1)} + \frac{2\mu^2\sigma^2}{n-1} + \frac{\sigma^6}{n}$.)

5. State and prove the Neyman-Pearson theorem.

6. Let X_1 and X_2 be independent, with $X_i \sim B(1, \theta)$. Consider testing that $\theta = 0.4$ against $\theta > 0.4$. Let $\Phi(X_1, X_2)$ be the nonrandomized test which rejects the null hypothesis if $X_1 = 1$.

   (a) Show that $T = X_1 + X_2$ is a sufficient statistic.

   (b) Show that $\Phi^*(T) = E[\Phi(X_1, X_2) \mid T] = 0$ if $T = 0$, $\Phi^*(T) = 1/2$ if $T = 1$, and $\Phi^*(T) = 1$ if $T = 2$.

國立政治大學應用數學系九十一學年度第一學期研究生學科考試試題

科目：數理統計

本卷共有6個題目，請選5題作答。

1. 設 (X, Y) 在三角形區域 $0 < x < y < 1$ 上具有均勻分配，

   (a) 求 $\mathrm{Var}(Y \mid X = 1/3)$ (10%)

   (b) 求 X/Y 的 p.d.f.   (10%)

2. 設 X, Y 是隨機變數，證明：

   (a) $EE(X \mid Y) = E(X)$ (5%)

   (b) $\mathrm{Var}(X) = \mathrm{Var}E(X \mid Y) + E\mathrm{Var}(X \mid Y)$ (15%)

3. (a) 設 $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} \Gamma(\alpha, \beta)$，求 $\alpha\beta$ 與 $\alpha^n\beta^n$ 的最佳不偏估計（best unbiased estimator）並且完整敘述所引用的重要定理。   (10%)

   (b) 設 $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} f(x; \theta) = \theta(1-\theta)^x$，$x = 0, 1, 2, \ldots$，$0 < \theta < 1$，求 $(1-\theta)/\theta$ 不偏估計的變異數之下限，又 $\overline{X}$ 是不是 $(1-\theta)/\theta$ 的有效不偏估計（efficient unbiased estimator）。   (10%)

4. 設 $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$，$\mu, \sigma^2$ 是未知參數，且 $0 < \alpha < 1$，

   (a) 導出 μ 的 $100(1-\alpha)\%$ 信賴區間，   (10%)

   (b) 導出 σ 的 $100(1-\alpha)\%$ 信賴區間。   (10%)

5. 設 $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(0, \theta^2)$，考慮 $H_0: \theta = 1$ 對 $H_a: \theta > 1$，推導出大小（size）為 α 的 UMP 檢定，$0 < \alpha < 1$，並完整敘述所引用的重要定理。   (20%)

6. (a) 敘述並證明 Rao-Blackwell 定理。   (10%)

   (b) 敘述並證明 Lehmann-Scheffé 定理。   (10%)

國立政治大學應用數學系九十一學年度第二學期學科考試試題

科目：數理統計

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選五題作答，但第三、四、六題必作。

一、 設 $X_1, X_2, \ldots, X_n$ 是 iid 連續隨機變數，且 X_i 的 pdf 為 f_X，cdf 為 F_X。令 $Y = \max(X_1, \ldots, X_n), Z = \min(X_1, \ldots, X_n)$，

    (a) 求 (Y, Z) 的結合 cdf。 （10分）

    (b) 求 (Y, Z) 的結合 pdf。 （10分）

    （以 f_X 或 F_X 表示出結果）

二、 設 $X_1, X_2, \ldots$ 是隨機變數序列，

    (a) 敘述 $X_n \xrightarrow{d} X$ 以及 $X_n \xrightarrow{p} b$ 的定義，其中 X 是隨機變數，b 是常數。

    （10分）

    (b) 若 $EX_n = \mu_n, Var(X_n) = \sigma_n^2$ 且 $\mu_n \longrightarrow a, \sigma_n^2 \longrightarrow 0$，證明 $X_n \xrightarrow{p} a$ （10分）

♠ 在下列三、四、五題中，設 $X_1, X_2, \ldots, X_n$ 是獨立隨機變數，$X_i \sim E(i\theta), \theta > 0$。

三、 (a) 分別求 θ 及 $\tau = \theta^2$ 的 MLE $\hat{\theta}$ 及 $\hat{\tau}$ （10分）

    (b) $\hat{\theta}$ 是不是 θ 的一致推定（consistent estimator）？ （10分）

    （必須敘述出所引用定理的內容）

四、 (a) $\hat{\theta}$ 是不是 θ 的有效推定（efficient estimator）？ （10分）

    (b) 求 $\eta = \theta^{-1}$ 的最佳不偏推定（best unbiased estimator）。 （10分）

    （必須敘述出所引用定理的內容）

五、 (a) 求 θ 的 $100(1 - \alpha)\%$ 信賴區間。 （10分）

    (b) 求 $\hat{\theta}_n$（基於 n 個觀測值）的漸近分配（asymptotic distribution）。

    （10分）

六、 設 $X_1, X_2, \ldots, X_{10} \overset{iid}{\sim} B(1, \theta)$。考慮 $H_0 : \theta = 0.4$ 對 $H_a : \theta > 0.4$

    (a) 導出大小（size）<u>恰為</u>0.05 的 UMP 檢定。 （10分）

    (b) 敘述出所引用定理的內容。 （10分）

    注意：$n = 10, \theta = 0.4$ 時，$X \sim B(n, \theta)$ 的累積機率如下：

x	0	1	2	3	4	5	6	7	8	9	10
P$(X \leq x)$	0.06	0.046	0.167	0.382	0.633	0.834	0.945	0.988	0.998	0.999	1

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國立政治大學應用數學系九十一學年度第二學期學科考試試題

國立政治大學應用數學系九十二學年度第一學期研究生學科考試試題

科目：數理統計

請任選五題作答：

1. 試解釋下列各名詞：

   (a) power function   (b) p-value   (c) unbiased test   (d) sufficient statistic

2. Let X_i be independent and $X_i \sim \text{Poisson}(i\theta), \theta > 0, i = 1, 2, \ldots, n$. Find the best unbiased estimator of θ.

3. Let $X_1, X_2, \ldots, X_n$ be independently uniformly distributed on $(0, \theta)$. Does there exist the best unbiased estimator of θ? If yes, find it.

4. State and prove the Neyman-Pearson Theorem on testing simple hypothesis verse another simple hypothesis.

5. Let X be a discrete random variable with density $f(x; \theta)$ giving in the following table:

x	0	1	2	3
$f(x; 0)$	.05	.05	.10	.80
$f(x; 1)$	.05	.20	.45	.30

   For testing $H_0 : \theta = 0$ against $H_1 : \theta = 1$, please list two different nonrandomized and two different randomized tests (any tests) of size .05. Compare these four tests (that is, is the first test more powerful than the second test, etc.?)

6. Consider a random sample of size n without replacement from a population of size N of which Np_1 are type A, Np_2 are type B, and $N(1 - p_1 - p_2)$ are type C. Let X_1 be the number of type A's in the sample, and let X_2 be the number of type B's.

   (a) What is the joint density function of X_1 and X_2?

   (b) Let $U = X_1 + X_2$. What are the mean and variance of U?

   (c) What is the conditional density function of $X_2 \mid U$?

1. Let X and Y be independent, and let $f_1(x) = e^{-x}$, $x > 0$, and $f_2(y) = e^{-y}$, $y > 0$.

   (a) Find the joint density of X and Y.

   (b) Find $P(X + Y \leq 1)$.

   (c) Find $P(X + Y \leq z)$ for all $z > 0$.

   (d) Let $Z = X + Y$. Find the density function of Z.

2. Define two statements A and B as follows:

   A: X and Y are independent;

   B: $E(X|Y = y) = E(X)$.

   (a) State the relationship between statements A and B. (i.e., A $\rightleftharpoons$ B, A $\overrightarrow{\nleftarrow}$ B, A $\underleftarrow{\nrightarrow}$ B, or A $\nrightleftharpoons$ B?)

   (b) Prove and/or disprove the relationship given in (a).

3. Let $(X_1, X_2, X_3, X_4) \sim M_4(n, (p_1, p_2, p_3, p_4))$. Let $U = X_2 + X_3$.

   (a) What is the distribution of (X_1, U)?

   (b) Show that the conditional distribution of X_1 given $X_2 = 2$ and $X_3 = 3$ is the same as the conditional distribution of X_1 given $U = 5$.

4. Let $X_i \sim N(i\theta, 1)$ and $X_1, X_2, \ldots, X_n$ are independent.

   (a) Find the MLE for θ and then show that this MLE is a consistent estimator of θ as n approaches infinity.

   (b) Both the MLE and the unbiased estimator of θ^2 are consistent estimators of θ^2.

5. Let $X_1, X_2, \ldots, X_{25}$ be a random sample from a normal distribution with mean μ and variance σ^2 unknown. Find the most powerful size-0.05 test for testing the null hypothesis $H_0 : \mu = \mu_0, \sigma^2 = \sigma_0^2$ against the alternative $H_1 : \mu = \mu_1, \sigma^2 = \sigma_0^2$.

6. Let X_1, X_2, and X_3 be independent, with $X_i \sim B(1, \theta)$. Consider testing that $\theta = 0.3$ against $\theta > 0.3$. Let $\Phi(X_1, X_2, X_3)$ be the nonrandomized test which rejects the null hypothesis if $X_3 = 1$.

   (a) Show that $T = X_1 + X_2 + X_3$ is a sufficient statistic.

   (b) Let $\Phi^*(T) = E[\Phi(X_1, X_2, X_3) \mid T]$. Find $\Phi^*(T)$.

1. Let X be a continuous random variable. Show that

$$EX = \int_0^\infty P(X > y)\, dy - \int_{-\infty}^0 P(X < y)\, dy.$$

2. Let X and Y be random variables. Show that

$$Var(Y) = E[Var(Y|X)] + Var[E(Y|X)].$$

3. Let (X, Y) have joint density function

$$f(x, y) = cxy,\ 0 < y < x < 1.$$

Find $Var(Y|X = 1/2)$.

4. Let $X_1, \ldots, X_n$ be independent, with $X_i \sim N(i\theta, 1)$.

   (a) Find the MLE of θ.

   (b) Find the best estimator of θ.

5. Let $X_1, \ldots, X_n$ be independent with $X_i \sim E(i\theta)$, $\theta > 0$. Find the lower bound for an unbiased estimator of θ^{-1}.

6. (a) State Neyman-Pearson Theorem.

   (b) Let $X_1, \ldots, X_n$ be independent, with $X_i \sim P(\theta)$. Find a UMP size-α test that $\theta = 1$ against $\theta > 1$.

1. Let the conditional density of Y given $\Lambda = \lambda$ be

$$f_1(y|\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}, \ y = 0, 1, 2, \ldots$$

and the density function of Λ be

$$f_2(\lambda) = e^{-\lambda}, \ \lambda > 0.$$

   (a) Find the marginal density of Y.

   (b) Find $E(Y|\Lambda = \lambda)$, $E(Y)$.

2. Suppose $X = (X_1, X_2, X_3)$ has joint density function

$$f(x_1, x_2, x_3) = \frac{3}{4\pi}, \ x_1^2 + x_2^2 + x_3^2 \le 1.$$

   Let $X_1 = U_1 \cos U_2 \sin U_3$, $X_2 = U_1 \sin U_2 \sin U_3$, $X_3 = U_1 \cos U_3$, $0 \le U_1 \le 1$, $0 \le U_2 \le 2\pi$, $0 \le U_3 \le \pi$.

   (a) Find marginal density functions for U_1, U_2, and U_3.

   (b) Find $P(1/4 \le U_1 \le 1/2, \pi/4 \le U_3 \le \pi/2)$.

3. (a) Let (X, Y) be random variables. Show that $E(X) = EE(X|Y)$ and $E(XY) = E(YE(X|Y))$.

   (b) Let (X, Y) be random variables with mean μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and correlation coefficient ρ. Suppose $E(X|Y) = aY + b$. Show that $b = \mu_X - a\mu_Y$, $a = \rho\sigma_X/\sigma_Y$.

4. Let $X_1, X_2, \ldots, X_n$ be independent with density function $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$.

   (a) Find the MLE of θ.

   (b) Find the best unbiased estimator of $1/\theta$.

   (c) Find an efficient estimator of $1/\theta$.

5. (a) State the following theorems: (i) Information inequality. (ii) Rao-Blackwell theorem.

   (b) Let $X \sim B(10, \theta)$. Consider testing $\theta = 0.3$ against $\theta \ne 0.3$ with the test

$$\Phi(X) = \begin{cases} 1, & \text{if } X > 6 \text{ or } X < 1, \\ 0.4, & \text{if } X = 6, \\ 0.2, & \text{if } X = 1, \\ 0, & \text{if } 1 < X < 6. \end{cases}$$

   Find the size of this test.

6. (a) State the following theorems: (i) Chebyshev's inequality (ii) Central limit theorem.

(b) Let $X_1, X_2, \ldots, X_{10}$ be independent, with $X_i \sim B(1, \theta)$. Find a UMP size-0.05 test that $\theta = 0.4$ against $\theta > 0.4$.

$$n = 10, \; p = 0.4, \; \sum_{x=0}^{r} b(x; n, p) \text{ 值表如下：}$$

r	0	1	2	3	4	5	6	7	8	9	10
	0.0060	0.0464	0.1673	0.3823	0.6331	0.8338	0.9452	0.9877	0.9983	0.9999	1.0000

1. Let (X, Y) be continuous random variables such that $f(x, y) = e^{-y}$, $0 < x < y < \infty$.

   (a) Find $E(Y \mid X = 1)$.

   (b) Find the joint moment generating function of (X, Y).

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the density

$$f(x; \theta) = \theta(1 + x)^{-(1+\theta)}, \ x > 0, \ \theta > 0.$$

   (a) Find the best unbiased estimator of $1/\theta$.

   (b) Find the information lower bound for the variance of an unbiased estimator of $1/\theta$.

3. (a) Let X_1, X_2, X_3 be independent with density

$$f(x_i; \alpha_i) = \frac{1}{\Gamma(\alpha_i)} x_i^{\alpha_i - 1} e^{-x_i}, \ x_i > 0.$$

   Let $U_1 = X_1 + X_2 + X_3, U_2 = X_2/U_1, U_3 = X_3/U_1$. Find the joint density of (U_1, U_2, U_3).

   (b) Let $X \sim N(0, 1)$ and $\Phi(x) = P(X \le x)$. Find the density of $\Phi(X)$.

4. (a) Suppose that for all n, T_n is an estimator of $\tau(\theta)$. Show that if $bias(T_n) \to 0$, $Var(T_n) \to 0$, then T_n is a consistent sequence of estimators.

   (b) Let $X_n \sim \Gamma(n, 1/n)$. Show that $X_n \xrightarrow{P} 1$.

5. Suppose $X_i \sim N(i\theta, 1)$, $i = 1, 2, 3$, are independent. Find the UMP size-0.05 test for testing $\theta = 2$ against $\theta > 2$. ($\sqrt{14} \doteq 3.741, \Phi(1.645) = 0.95$ where $\Phi(x) = \int_{-\infty}^{x} (1/\sqrt{2\pi}) e^{-t^2/2} \, dt$)

6. Suppose X is a discrete random variable taking on the values $1, 2, 3$, and 4, and θ takes on the values $-1, 0$, and 1. Suppose also that the density of X is giving in the following table:

x	1	2	3	4
$f(x; -1)$	0.53	0.30	0.00	0.17
$f(x; 0)$	0.60	0.20	0.10	0.10
$f(x; 1)$	0.60	0.22	0.18	0

Consider testing that $\theta = 0$ against $\theta \ne 0$.

   (a) Show that the 0.2 LRT for this problem rejects the null hypothesis if $X = 3$ or 4.

   (b) Show that the test which rejects the null hypothesis if $X = 2$ is a size-0.2 test which is more powerful than the LRT.

1. (a) Consider a Poisson process with rate λ. Let X be the time till the second arrival. Find the density of X.

   (b) Suppose that we sample with replacement from a population of size N with proportion of successes p. Let V be the number of failures before the r-th success. Find the moment-generating function of V.

2. Let Z and U be independent, $Z \sim N(0,1)$, and $U \sim \chi_k^2$. Let $T = Z/\sqrt{U/k}$ and $W = U$.

   (a) Find the joint density of T and W.

   (b) Find the marginal density of T.

3. Let $X_1, \ldots, X_n$ be independent, with $X_i \sim N(\mu, \sigma^2)$. Find the size-$\alpha$ LRT for testing that $\sigma^2 = c^2$ against $\sigma^2 > c^2$, and show that it rejects the null hypothesis if $U > \chi_{n-1}^2(\alpha)$, where $U = (n-1)S^2/c^2$.

4. Let $X_1, \ldots, X_n$ be independent, with $X_i \sim E(i\theta)$.

   (a) Find the MLE of θ.

   (b) Find a $(1-\alpha)$ confidence interval for θ.

5. (Problem 4 continued)

   (a) Find the lower bound for an unbiased estimator of $1/\theta$.

   (b) Find the best unbiased estimator of θ^2.

6. (a) Let X be a discrete random variable having the following density function:

x	0	1	2	3	4	5
$f(x;0)$	.05	.05	.10	.10	.20	.50
$f(x;1)$	.10	.15	.25	.15	.25	.10

   Find the most powerful .15 test that $\theta = 1$ against $\theta = 0$.

   (b) State and prove Neymann-Pearson Theorem for the continuous case.

1. Let X and Y be independent, and let $f_1(x) = e^{-x}$, $x > 0$, and $f_2(y) = e^{-y}$, $y > 0$. Let $U = X + Y$, $V = X/(X + Y)$. Find

   (a) the joint density of U and V,

   (b) the conditional density of U given $V = v$, and

   (c) the marginal density of U.

2. Let $X_1, \ldots, X_n$ be independent, and let $X_i \sim N(\mu, \sigma^2)$, $n > 1$.

   (a) What are the distributions of the sample mean $\overline{X}$ and the sample variance S^2?

   (b) Are $\overline{X}$ and S^2 independent?

3. Let $X_1, X_2, \ldots$ be a sequence of independent random variables with X_i uniformly distributed on the interval $(0, 1)$. Find the limiting distribution of $W_n = n(1 - \max_{i \leq n} X_i)$.

4. Let X_i be independent Poisson distribution with mean $i\theta$, where $i = 1, 2, \ldots, n$. Now, let $P = \sum_{i=1}^{n}(X_i/a_n)$ and $T = P^2 - P/a_n$, where $a_n = \sum_{i=1}^{n} i$. Prove or disprove that

   (a) P is a consistent estimator of θ.

   (b) T is a consistent estimator of θ^2.

5. State and prove the Neyman-Pearson theorem.

6. We say that a statistic T has a complete family of distributions if $E_\theta h(T) = 0$ for all θ implies that $h(T) \equiv 0$. Use this definition to show that $\overline{X}$ has a complete family of distributions and is a complete sufficient statistic, where $X_1, \ldots, X_n$ are independent and $X_i \sim N(\theta, 1)$.

國立政治大學應用數學系九十五學年度第二學期研究生學科考試試題

科目：數理統計

1. Let (X, Y) have joint density $f(x, y) = kx^2y^3$, $0 < y < x < 1$.

   (a) Find the marginal density of Y.

   (b) Find $Var(X \mid Y = 2/3)$.

2. (a) Consider a Poisson process with rate λ. Let T be the time till the third arrival. Find the density of T.

   (b) Find the moment generating function of T.

3. Let X_1, X_2, X_3 be independent and $X_i \sim P(m_i)$. Let $Y = X_1 + X_2 + X_3$, and $g = m_1 + m_2 + m_3$. Show that $(X_1, X_2) \mid Y \sim T(Y, (m_1/q, m_2/q))$.

4. Let $(X, Y) \sim T(n, (\theta^2, 2\theta(1 - \theta))$.

   (a) Find the lower bound for an unbiased estimator of θ.

   (b) Find the best unbiased estimator of θ.

5. Let $X_1, X_2, \ldots, X_n$ be independent, with $X_i \sim E(i\theta)$. Show that the LRT for testing that $\theta = c$ against $\theta < c$ rejects the null hypothesis if $U = 2nR/c$ is too small, where $R = \sum_{i=1}^{n} X_i/n_i$.

6. Let $X_1, X_2, \ldots, X_n$ be independent, with $X_i \sim E(i\theta)$.

   (a) Find a $(1 - \alpha)$ confidence interval for θ, where $0 < \alpha < 1$.

   (b) Find a UMP size-α test for testing $\theta = c$ against $\theta > c$.

■ $(X_1, X_2) \sim T(n, (\theta_1, \theta_2)) \Leftrightarrow f(x_1, x_2) = \frac{n!}{x_1!x_2!(n-x_1-x_2)!}\theta^{x_1}\theta_2^{x_2}(1 - \theta_1 - \theta_2)^{n-x_1-x_2}$, $x_1, x_2 \geq 0$, $x_1 + x_2 \leq n$.

■ $X \sim P(\lambda) \Leftrightarrow f(x) = e^{-\lambda}\lambda^x/x!$, $x = 0, 1, 2, \ldots$

■ $X \sim E(\theta) \Leftrightarrow f(x) = (1/\theta)e^{-x/\theta}$, $x > 0$.

1. Let (X, Y) be jointly uniformly distributed on the triangle $0 < x < y < 1$. Let $U = X/Y$.

   (a) Find the marginal densities of X and Y.

   (b) Find the density of U.

2. Let (X, Y, Z) have joint moment-generating function $M(r, s, t) = (1 - r)^{-2}(1 + r - 2s)^{-3}(1 + 2r - s + 3t)^{-1}$.

   (a) Find $Cov(X, Y)$.

   (b) Find the joint moment-generating function of $U = 2X - Y$ and $V = Z - Y$. Are U and V independent?

3. (a) Let X, Y be random variables. Show that $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$.

   (b) Let (X, Y) have joint density $f(x, y) = cxy^2$, $0 < x < y < 1$. Find c and $Var(X|Y = 2/3)$.

4. Let $X_1, X_2, \ldots, X_n$ be independent with $X_i \sim E(i\theta)$, and let $R = \sum_{i=1}^n X_i/n_i$.

   (a) Find an unbiased estimator of θ^{-1}.

   (b) Is this estimator efficient?

5. (a) State the Neyman-Pearson theorem and prove it for the continuous case.

   (b) Let $X_1, \ldots, X_n$ be independent with $X_i \sim B(1, \theta)$. Find a UMP size-0.1 test that $\theta = 0.4$ against $\theta < 0.4$ when $n = 12$.

6. (a) Let $(X, Y) \sim T(n, (\theta^2, 2\theta(1 - \theta)))$. Find the lower bound for an unbiased estimator of θ^2.

   (b) Let X be a positive continuous random variable. Show that $E(X) = \int_0^\infty P(X > y) \, dy$.

Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

						p					
n	r	.10	.20	.25	.30	.40	.50	.60	.70	.80	.90
12	0	.2824	.0687	.0317	.0138	.0022	.0002	.0000			
	1	.6590	.2749	.1584	.0850	.0196	.0032	.0003	.0000		
	2	.8891	.5583	.3907	.2528	.0834	.0193	.0028	.0002	.0000	
	3	.9744	.7946	.6488	.4925	.2253	.0730	.0153	.0017	.0001	
	4	.9957	.9274	.8424	.7237	.4382	.1938	.0573	.0095	.0006	.0000
	5	.9995	.9806	.9456	.8821	.6652	.3872	.1582	.0386	.0039	.0001
	6	.9999	.9961	.9857	.9614	.8418	.6128	.3348	.1178	.0194	.0005
	7	1.0000	.9994	.9972	.9905	.9427	.8062	.5618	.2763	.0726	.0043
	8		.9999	.9996	.9983	.9847	.9270	.7747	.5075	.2054	.0256
	9		1.0000	1.0000	.9998	.9972	.9807	.9166	.7472	.4417	.1109
	10				1.0000	.9997	.9968	.9804	.9150	.7251	.3410
	11					1.0000	.9998	.9978	.9862	.9313	.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000
13	0	.2542	.0550	.0238	.0097	.0013	.0001	.0000			
	1	.6213	.2336	.1267	.0637	.0126	.0017	.0001	.0000		
	2	.8661	.5017	.3326	.2025	.0579	.0112	.0013	.0001		
	3	.9658	.7473	.5843	.4206	.1686	.0461	.0078	.0007	.0000	
	4	.9935	.9009	.7940	.6543	.3530	.1334	.0321	.0040	.0002	
	5	.9991	.9700	.9198	.8346	.5744	.2905	.0977	.0182	.0012	.0000
	6	.9999	.9930	.9757	.9376	.7712	.5000	.2288	.0624	.0070	.0001
	7	1.0000	.9980	.9944	.9818	.9023	.7095	.4256	.1654	.0300	.0009
	8		.9998	.9990	.9960	.9679	.8666	.6470	.3457	.0991	.0065
	9		1.0000	.9999	.9993	.9922	.9539	.8314	.5794	.2527	.0342
	10			1.0000	.9999	.9987	.9888	.9421	.7975	.4983	.1339
	11				1.0000	.9999	.9983	.9874	.9363	.7664	.3787
	12					1.0000	.9999	.9987	.9903	.9450	.7458
	13						1.0000	1.0000	1.0000	1.0000	1.0000
14	0	.2288	.0440	.0178	.0068	.0008	.0001	.0000			
	1	.5846	.1979	.1010	.0475	.0081	.0009	.0001			
	2	.8416	.4481	.2811	.1608	.0398	.0065	.0006	.0000		
	3	.9559	.6982	.5213	.3552	.1243	.0287	.0039	.0002		
	4	.9908	.8702	.7415	.5842	.2793	.0898	.0175	.0017	.0000	
	5	.9985	.9561	.8883	.7805	.4859	.2120	.0583	.0083	.0004	
	6	.9998	.9884	.9617	.9067	.6925	.3953	.1501	.0315	.0024	.0000
	7	1.0000	.9976	.9897	.9685	.8499	.6047	.3075	.0933	.0116	.0002
	8		.9996	.9978	.9917	.9417	.7880	.5141	.2195	.0439	.0015
	9		1.0000	.9997	.9983	.9825	.9102	.7207	.4158	.1298	.0092
	10			1.0000	.9998	.9961	.9713	.8757	.6448	.3018	.0441
	11				1.0000	.9994	.9935	.9602	.8392	.5519	.1584
	12					.9999	.9991	.9919	.9525	.8021	.4154
	13					1.0000	.9999	.9992	.9932	.9560	.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000