國立政治大學 應用數學系 **103** 學年度第 一 學期　學科 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM　Page

考試科目 Course	作業研究	開課系級 Dept, & Class		日　期 Date, Period	9 月 22 日 第一節	試題編號 Course No.	

本試卷共有 6 題，碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Consider the following problem:

$$\begin{aligned} &\min\ x_1 - 2x_2 \\ &\text{s.t}\quad x_1 - x_2 \ge 2,\quad x_2 \le 3 \\ &\quad -x_1 + x_2 \ge 1,\quad x_1, x_2 \ge 0 \end{aligned}$$

The slack variables x_3, x_4, and x_5 are introduced and the artificial variables x_6 and x_7 are incorporated in the first two constraints. The problem was solved by the big-M method and the final tableau is as follows:

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1								
x_4	0					1	0	−1	
x_2	0					1	0	0	
x_5	0					1	−1	0	

Unfortunately, some of the data are missing, fill in the missing data. Show your calculations. (20%)

2. Consider the Toyco problem, the model and the final tableau are as follows:

$$\begin{aligned} &\max\ z = 3x_1 + 2x_2 + 5x_3 \\ &\text{s.t.}\quad x_1 + 2x_2 + x_3 \le 430 \quad \text{(Operation 1)} \\ &\quad 3x_1 + 2x_3 \le 460 \quad \text{(Operation 2)} \\ &\quad x_1 + 4x_2 \le 420 \quad \text{(Operation 3)} \\ &\quad x_1, x_2, x_3 \ge 0 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	4	0	0	1	2	0	1350
x_2	−1/4	1	0	1/2	−1/4	0	100
x_3	3/2	0	1	0	1/2	0	230
x_6	2	0	0	−2	1	1	20

a. Suppose that the company reduce the unit times on operation 1, 2, and 3 for toy trains (x_1) from the current level of 1, 3, and 1 minutes to .5, 1, and .5 minutes, respectively. The revenue per unit is change to \$4. Determine the new optimum solution. (10%)

b. Suppose that a new toy (fire engine) requires 3, 2, and 4 minutes, respectively on operations 1, 2, and 3. If the revenue of the new toy is \$8, determine the optimal solution. (10%)

3. Solve the following problem by the bounded algorithm: (20%)

$$\begin{aligned} &\max\ z = 3x_1 + 2x_2 - 2x_3 \\ &\text{s.t.}\quad 2x_1 + x_2 + x_3 \le 8 \\ &\quad x_1 + 2x_2 - x_3 \ge 3 \\ &\quad 1 \le x_1 \le 3, 0 \le x_2 \le 3, 2 \le x_3 \end{aligned}$$

本考試：□ 不需使用簡易計算機，☑ 使用簡易計算機　　請出題老師勾選，謝謝！

命題老師：(Teacher)	(簽章) 年 月 日 (Signature & date)	試題隨卷繳交

國立政治大學 應用數學系 **102** 學年度第 一 學期　學科 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM　Page

考試科目 Course	作業研究	開課系級 Dept, & Class		日　期 Date, Period	9 月 22 日 第一節	試題編號 Course No.	

4. A business executive must make the four round trips between Dallas and Atlanta listed in the following table. The price of a round-trip ticket from Dallas is $400. A discount of 25% is granted if the dates of arrival and departure of a ticket span a weekend (Saturday and Sunday). If the stay in Atlanta lasts more than 21 days, the discount is increased to 30%. A one-way ticket between Dallas and Atlanta (either direction) costs $250. How should the executive purchase the tickets?

Departure date from Dallas	Return date to Dallas
Monday, June 3	Friday, June 7
Monday, June 10	Wednesday, June 12
Monday, June 17	Friday, June 21
Tuesday, June 25	Friday, June 28

a. Formulate this problem as an assignment problem. (10%)

b. Solve this problem by using Hungarian method. List the associate dual solution. (10%)

5. Jobco Shop has 4 outstanding jobs to be processed on a single machine. The following table provides processing times and due time. All time are in days and due time is measured from time 0. Job 4 must be preceded job 3.

Job	Processing time	Due time
1	10	16
2	13	25
3	22	40
4	17	36

a. The objective is to process all 4 jobs in the shortest possible time. Formulate the model as an ILP. (10%)

b. The objective is to minimum the maximum lateness. Formulate the model as an ILP. (10%)

6. Every year, the gardener will check the soil condition. The soil condition is one of the three states: (1) good, (2) fair, and (3) poor. The soil condition for the new season can be describe as a Markov Chain $\mathbf{P}$. The garner can alter the transition probabilities $\mathbf{P}$ by using the fertilizer to boost soil condition. In this case, the transition matrix becomes $\mathbf{P}_1$ as follows:

$$\mathbf{P} = \begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{P}_1 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$$

a. Find the steady-state probabilities for the $\mathbf{P}_1$. (5%)

b. Consider the gardener Markov chain with fertilizers. What is the first passage time from states 2 (fair) to state 1 (good) in 3 transitions? (5%)

c. Consider the gardener Markov chain with fertilizers. What is the mean first passage time from states 2 (fair) to state 1 (good)? (5%)

d. Consider the gardener problem without fertilizer. What is the expected number of seasons that the soil will become poor given that the condition is good now? (5%)

本考試：□ 不需使用簡易計算機，☑ 使用簡易計算機　　請出題老師勾選，謝謝！

命題老師：(Teacher)	(簽章) 103 年 8 月 27 日 (Signature & date)	試題隨卷繳交