(榮譽第一)

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國立政治大學 應用數學系 101 學年度第 一 學期　學科 考試試題

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考試科目 Course	實變函數論	開課系級 Dept, & Class	研究所	日期 Date, Period	101 年 9 月 24 日 上午 9:00~12:00	試題編號 Course No.	

本試卷共有 6 個題目，

碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。

博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

一. Show that a subset E of $\mathbb{R}^n$ is Lebesgue measurable if and only if $\forall \varepsilon > 0$, $\exists$ an open set $G_\varepsilon \ni E \subseteq G_\varepsilon$ and $m_n(G_\varepsilon - E) < \varepsilon$, where m_n is the n-dimensional Lebesgue measure on $\mathbb{R}^n$.

二. Let $1 \le p < \infty$, $f, f_n \in L^p(X, \mathcal{S}, \mu)$, $n \geq 1$. If $f_n \to f$ a.e. on X and $\|f_n\|_p \to \|f\|_p$ as $n \to \infty$, then $\|f_n - f\|_p \to 0$ as $n \to \infty$.

三. Show that every compact metric space is separable.

四. Let V be a normed linear space. Show that the dual space V^* is a Banach space.

五. In $L^p(X, \mathcal{S}, \mu)$, does the Minkowski's inequality hold for all $0 < p \le \infty$?

六. State and prove the Lebesgue Monotone Convergence Theorem.

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命題老師 (Teacher)	簽章) 101 年 9 月 20 日 (Signature & date)	試題隨卷繳交

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考試科目 Course	數理統計	開課系級 Dept, & Class	研究所	日期 Date, Period	101 年 9 月 24 日 上午 9:00~12:00	試題編號 Course No.	

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博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

To earn your credits, you must show your work.

You may use the calculator and the information at the end of page 2.

1. Suppose that a lie detector test has the following properties. If the suspect is telling the truth, the lie detector will correctly say so with probability 0.9; if the suspect is lying, it will correctly identify this with probability 0.99. If 95% of the people are telling the truth, find the probability that a person is actually lying when the test says he or she is. What can you tell for your probability in practice?

2. Consider a Poisson process with rate $\lambda = 3$. Let X be the time till the first arrival.

   (a) Find P (X > 0.7). (b) Find the density function of X.

3. Let $\mathbf{X} = (X_1, X_2, X_3)'$ have a trivariate normal distribution with means 6, 4, and 2 and variances 16, 25, and 64, and with cov $(X_1, X_2)=6$ and $\text{cov}(X_1, X_3)=\text{cov}(X_2, X_3)=0$.

   Define $Y_1=2X_1+3X_2+X_3+1$ and $Y_2=4X_1+X_3+3$.

   (a) What is the joint distribution of $\mathbf{Y} = (Y_1, Y_2)'$?

   (b) What is the correlation coefficient between Y_1 and Y_2?

   (c) What is the moment-generating function of $\mathbf{Y}$?

   (d) Find the conditional distribution of Y_2 given $Y_1=29$.

本考試：

命題老師 (Teacher)	（簽章）(Signature & date) 2012年 9 月 20 日	試題隨卷繳交

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考試科目 Course	數理統計	開課系級 Dept, & Class	研究所	日期 Date, Period	101 年 9 月 24 日 上午 9:00~12:00	試題編號 Course No.	

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4. Assume that X_i are independent and $X_i \sim N(i\theta, 1)$, $i=1, 2, \cdots, n$, θ is a real number.

   (a) Find S, the MLE of θ.

   (b) Find the MLE of θ^3 in terms of S in (a).

   (c) Is S an efficient unbiased estimator of θ? Why?

   (d) Let $I_n(\theta)$ be the Fisher information based on n observations. Find the distribution of $[I_n(\theta)]^{0.5}(S-\theta)$.

5. Assume that X_i are independent and $X_i \sim N(i\theta, \sigma^2)$, $i=1, 2, \cdots, n$, where θ and $\sigma>0$ are unknown parameters.

   (a) Find a pivotal quantity and a $(1-\alpha)$ confidence interval for θ.

   (b) Find a pivotal quantity and a $(1-\alpha)$ confidence interval for σ^2.

   (Hint: You may use the following result. $T^2 = \frac{\sum_{i=1}^{n}(X_i - iS)^2}{(n-1)} \sim \Gamma\left(\frac{n-1}{2}, \frac{2\sigma^2}{n-1}\right)$, where S is the same as that in Question 4(a).)

6. State and prove the Rao-Blackwell theorem.

Note: If a random variable X has a Poisson distribution with mean m>0, then X has the density function $f(x)=\frac{e^{-m}m^x}{x!}$, $x=0, 1, 2, \ldots$

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命題老師 (Teacher)	(簽章) 2012 年 9 月 20 日 (Signature & date)	試題隨卷繳交

國立政治大學 應用數學系 101 學年度第 一 學期 學科 考試試題
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考試科目 Course	作業研究	開課系級 Dept, & Class		日期 Date, Period	9 月 24 日 第一節	試題編號 Course No.	

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博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Given the problem as follows:

$$\begin{aligned} \min \quad & z = 5x_1 + 3x_2 - 2x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 \ge 8 \\ & 2x_1 - 5x_2 + x_3 \ge 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

   a. Solve the problem by using *big-M* method. (10%)
   b. Solve the problem by dual simplex method. (10%)

2. Write the dual for the following primal problem: (20%)

   a.

$$\begin{aligned} \max \quad & z = 2x_1 + 2x_2 + 3x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 \le 20 \\ & 3x_1 + 4x_2 + 2x_3 = 8 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

   b.

$$\begin{aligned} \min \quad & z = 3x_1 - 2x_2 + 4x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 \ge 20 \\ & 3x_1 + 2x_2 - 2x_3 \ge 8 \\ & x_1 + 2x_2 + x_3 \le 15 \\ & x_1, x_2 \ge 0, x_3 \text{ unrestricted} \end{aligned}$$

3. Study the variation in the optimal solution of the following parameterized LP, given $t \ge 0$. (20%)

$$\begin{aligned} \max \quad & z = 3x_1 + 2x_2 + 5x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \le 40 + 2t \\ & 3x_1 + 2x_3 \le 60 - t \\ & x_1 + 4x_2 \le 30 + t \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

At $t = t_0 = 0$, the optimal solution is as follows:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	4	0	0	1	2	0	160
x_2	−1/4	1	0	1/2	−1/4	0	5
x_3	3/2	0	1	0	1/2	0	30
x_6	2	0	0	−2	1	1	10

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命題老師 (Teacher)	（簽章）(Signature & date) 年 月 日	試題隨卷繳交

國立政治大學 應用數學系 101 學年度第 一 學期 學科 考試試題

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4. Cars are shipped from three distribution centers to four dealers. The shipping cost is based on the mileage between the source and destinations and is independent of whether the truck makes the trip with partial or full loads. The table summarizes the mileage between the distributions centers and the dealers together with the monthly supply and demand figures in number of cars. A full truckload includes 20 cars. The transportation cost per truck mile is $10.
   a. Formulate the associate transportation model. (10%)
   b. Determine the optimal shipping schedule. (10%)

	Dealer				
	1	2	3	4	Supply
Center 1	100	70	100	140	**250**
Center 2	50	60	60	40	**200**
Center 3	40	80	80	120	**160**
Demand	**100**	**200**	**150**	**160**	

5. For the upcoming planting season, Farmer McCoy can plant corn (a_1), plant wheat (a_2), plant soybeans (a_3), or use the land for grazing (a_4). The payoffs associated with the different actions are influenced by the amount of rain: heavy rainfall (s_1), moderate rainfall (s_2), light rainfall (s_3), or drought season (s_4). The payoff matrix (in thousands of dollars) is estimated as follows:

	s_1	s_2	s_3	s_4
a_1	−20	55	30	60
a_2	60	50	35	20
a_3	−50	30	45	−10
a_4	40	15	35	10

   Recommend a course of action for Farmer McCoy (based on each of the four criteria of decision under uncertainty, for Hurwicz method using $\alpha = .5$). (20%)

6. On a sunny day, MiniGolf can gross $2000 in revenues. If the day is cloudy, revenues drop by 20%. A rainy day will reduce revenue by 80%. If today's weather is sunny, there is an 80% chance it will remain sunny tomorrow with no change of rain. If it is cloudy, there is a 20% change that tomorrow will be rainy and 30% change will be sunny. Rain will continue through the next day with a probability of 0.8, but there is a 10% change it may be sunny.
   a. Represent the weather changing as a Markov chain. (5%)
   b. If Monday is sunny, what is the probability that the first rainy day is on Thursday? (5%)
   c On average it will take how many days that the weather will becomes rainy day. (10%)

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考試科目 Course	微分方程式	開課系級 Dept, & Class	研究所	日期 Date, Period	101 年 9 月 24 日 上午 9:00~12:00	試題編號 Course No.	

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博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Discuss the existence and uniqueness of the solutions of the following initial value problems:
   (a) $y' = |x+y|, y(0) = 1$
   (b) $y' = \sqrt{|y|}, y(0) = 0$

2. Let $f: \mathbb{R}^n \to \mathbb{R}^n$ satisfy $|f(x) - f(y)| \le |x - y|$ for all $x, y \in \mathbb{R}^n$. Prove that the initial value problem $x' = f(x),\ x(0) = x_0$ has a unique solution on $(-\infty, \infty)$.

3. Let $p(t)$ and $q(t)$ be continuous functions in $[t_0, \infty)$. Suppose that all solutions of $x'' + p(t)x = 0$ are bounded in $[t_0, \infty)$. Show that all solutions of $x'' + (p(t) + q(t))x = 0$ are bounded in $[t_0, \infty)$ provided that $\int_{t_0}^{\infty} |q(t)|\, dt < \infty$.

4. In (a) and (b), determine whether the trivial solution is stable, asymptotically stable or unstable.
   (a) $\begin{aligned} x_1' &= \ln(1 - x_3) \\ x_2' &= \ln(1 - x_1) \\ x_3' &= \ln(1 - x_2) \end{aligned}$　(b) $y''' + 3y'' - 4y' + y^2 = 0$

5. Show that the system
$$\frac{dx}{dt} = x\left(\lambda - (x^2 + 2y^2)\right) + 3y,$$
$$\frac{dy}{dt} = y\left(\lambda - (x^2 + 2y^2)\right) - 3x$$
has a limit cycle for $\lambda > 0$.

6. Let ϕ and ψ be linearly independent solutions of the second-order linear equation $(p(t)x')' + q(t)x = 0$ on (a,b), where $p(t)$ and $q(t)$ are continuous in (a,b). Show that the zeros of ϕ and ψ separate each other.

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1. Find a simple group of order 20 or show that it is impossible.

2. Let G be a group of order 8.
   (a) Suppose that G is abelian. Find all possible group structures (up to isomorphism) for the group G.
   (b) Find ONE explicitly example for G such that G is not abelian.

3. Let R be a commutative ring with unity. Then M is a maximal ideal of R if and only if R/M is a field.

4. Let G be a group. For each g in G, define the function

$$\phi_g : G \to G$$

by $\phi_g(x) = gxg^{-1}$, for all $x \in G$. Let

$$\mathrm{Inn(G)} = \{\phi_g \mid \mathrm{g} \in \mathrm{G}\}.$$

   (a) For each $g \in G$, show that ϕ_g is an automorphism.
   (b) Show that Inn(G) is a group (under the operation of function composition).

5. Let G be a group and let $Z(G)$ be the center of G. Suppose that $G/Z(G)$ is cyclic. Show that G is abelian.

6. Show that for any prime p, the p-th cyclotomic polynomial

$$f(x) = x^{p-1} + x^{p-2} + \cdots + x + 1$$

is irreducible over $\mathbf{Q}$.

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