資格考試參考資料－實變函數論
Real Analysis Qualifying Exam Syllabus

I. Euclidean n-space R^n

1. Elementary point set topology.
2. Riemann and Improper Riemann integral.
3. Borel and Lebesgue measures.
4. Lebesgue integral.

II. Measure and Integration Theory

1. Measure spaces, Completion of measure space.
2. Measurable functions.
3. Integration theory.
4. Convergence theorem.
5. Signed measures.
6. Radon-Nikodym theorem.
7. L^p -spaces.
8. Outer measure.
9. Lebesgue integral
10. Lebesgue-Stieltjes integral.
11. Product measures.
12. Fubini theorem.

III. Abstract Spaces

1. Metric space and its elementary properties.
2. Ascoli-Arzela theorem.
3. Arzela-Ascoli theorem
4. Abstract topological spaces and its elementary properties.
5. Stone-Weierstrass theorem.
6. Normed linear spaces, Banach spaces, Hibert spaces and their related properties.
7. Normed spaces
8. Hölder and Minkowski inequalities
9. Metric spaces and its elementary properties
10. Banach spaces, Hilbert spaces and their related properties

[References]

1. L. Royden, Real Analysis
2. R. Wheeden & A. Zygmund, Measure and Integral, CRC Pure and Applied Mathematics, 2nd edition

資格考試參考資料－微分方程式
Differential Equations Qualifying Exam Syllabus

I. Fundamental Theory

1. Existence of solutions
2. Uniqueness of solutions
3. Continuity of solutions with respect to parameters
4. Comparison theorems

II. Linear Systems

1. Homogeneous and nonhomogeneous systems
2. Linear system with constant coefficients
3. Linear system with periodic coefficients (Floquet theory)
4. Oscillation theorems
5. Asymptotic behavior of solutions

III. Stability

1. Fundamental stability theorems
2. Instability theorem
3. Lyapunov stability

IV. Periodic solutions of systems

1. Poincaré-Bendixon theory (n=2)
2. Periodic solutions of nonhomogeneous linear systems

V. Second order linear differential equations

1. Boundedness theorems
2. Asymptotic behavior of solutions

[References]

1. R. Bellman, Stability Theory of Differential Equations
2. Ravi P. Agarwal and Donal O’Regan, An Introduction to Ordinary Differential Equations

資格考試參考資料－數值方法
Numerical Analysis Qualifying Exam Syllabus

1. Systems of Linear Equations:
   Matrix algebra, The LU and Cholesky factorizations, Pivoting and construction and algorithm, Norms and the analysis errors, Neumann series and iterative refinement, solution of equations by iterative methods, steepest descent and conjugate gradient methods

2. Approximating functions:
   Polynomial Interpolation, orthogonal polynomials, Spline Interpolation, B-Splines, Taylor Series, trigonometric interpolation and fast Fourier transform

3. Nonlinear equations:
   Bisection method, Newton’s method, Secant method, Fixed points and Functional iteration, computing zeros of polynomials, Homotopy and continuation method

4. Eigenvalue Problems:
   Jordan Normal form, Schur Normal form, Hermitian and Normal matrix, Reduce matrix to simpler form, compute of eigenvalue and eigenvectors

5. Numerical Differentiation and Integration:
   Numerical differentiation and Richardson extrapolation, Gaussian quadrature, Romberg integration, adaptive quadrature, Sard’s theory of approximating functions

6. Numerical solution of ordinary differential equations:
   Existence and Uniqueness of solutions, Taylor-series method, Runge-Kutta Methods, Multi-step method, Finite-difference methods

7. Numerical solution of partial differential equations:
   Explicit methods and Implicit methods, Finite-Difference Methods, Galerkin and Ritz Methods. Multigrid Method

[References]

1. Stoer and Bulirsch, Introduction to Numerical Analysis.

2. Kincaid and Cheney, Numerical Analysis.

資格考試參考資料－數理統計
Mathematical Statistics Qualifying Exam Syllabus

I. Probability models

1. Sample Spaces, Events
2. Probability Axioms
3. Conditional Probability and Independence

II. Random Variables, Random Vectors and Their Distributions

1. Density Functions, Distribution Functions
2. Bivariate distributions, Multivariate Distributions
3. Expectation, Moments of a Distribution, Moment Generating Functions
4. Conditional Expectation
5. Distributions of Functions of Random Variables

III. Some Parametric Families

1. Normal Distribution
2. Distributions Associated with Bernoulli Trials
3. Distributions Associated with Poisson Process
4. Distributions Associated with Normal Distribution
5. Multinomial Distributions
6. Bivariate Normal Distribution

IV. Asymptotic Distributions

1. Convergence in Probability and Distribution
2. The Weak Law and the Central Limit Theorem
3. Continuous Functions and Slutzky's Theorem

V. Estimation

1. Maximum Likelihood Estimators
2. Unbiased Estimators, Consistent Estimators, Efficient Estimators
3. Confidence Intervals

VI. Optimal Tests

1. Randomized Tests, Nonrandomized Tests
2. Power Function
3. Uniformly Most Powerful Tests
4. Likelihood Ratio Tests

VII. Sufficient Statistics

1. Definition and Criteria for Sufficiency
2. Minimal and Complete Sufficient Statistics
3. Uniformly Minimal Variance Unbiased Estimators

[References] Steven F. Arnold, Mathematical Statistics

資格考試參考資料－作業研究
Operations Research Qualifying Exam Syllabus

1. Mathematical modeling
   operations research modeling approach, maximization & minimization problem, classic application forms: allocation & blending models, operations planning & shift scheduling models
2. Linear programming and its dual
   the simplex method, duality theorems, complementary slackness conditions, sensitivity analysis, parametric programming
3. Transportation problem
   balanced /unbalanced transportation problem, transshipment problem
4. Advanced LP techniques
   upper-bounded simplex, column generation method, Karmarkar's method
5. Network flow problems
   network simplex method, maximal flow /minimal cost flow problem
6. Dynamic programming
   EOQ inventory models, probabilistic inventory models
7. Integer programming
   branch and bound method, cutting plane algorithm
8. Markov chains
   classification of states, steady-state probabilities, the hitting time
9. Queueing models
   M/M/1, M/G/1, G/M/1, Er/Er/1, Ph/Ph/1 models
10. Queueing networks
    Jackson networks and their applications
11. Nonlinear programming
    Lagrange multipliers, K-K-T conditions, Unconstrained problems

[References]

1. Hamdy A. Taha, “Operations Research, An Introduction” Pearson Education, 2007.
2. F.S. Hillier and G. J. Lieberman, “Introduction to Operations Research” McGraw- Hill Science, 2004.

資格考試參考資料－應用代數
Applied Algebra Qualifying Exam Syllabus

I. Group Theory:

1. Basic materials in group theory, such as subgroups, three isomorphism theorems, Jordan-Hölder theorem, Lagrange's theorem, Cayley's theorem, Sylow's theorems and applications, fundamental theorem for finitely generated abelian groups
2. Linear groups $(GL(n, F)$ and $SL(n, F))$
3. Groups actions
4. Symmetric groups, free groups, nilpotent and solvable groups, simple groups

II. Rings and Modules:

1. Basic materials in ring theory, such as ideals, quotient rings, ring homomorphisms, polynomial rings, Euclidean domains, principal ideal domains, unique factorization domains, Gauss's lemma, local rings, localization, Nakayama's lemma, integral ring extensions. Dedekind domains, matrix rings, division rings
2. Prime ideals and maximal ideals, Chinese remainder theorem,
3. Chain conditions, Noetherian rings
4. Basic materials in module theory, such as modules, module homomorphisms, quotient modules, free modules
5. Finitely generated modules over a PID
6. Torsion modules, primary components, invariance theorem

III. Field Theory:

1. Field extensions, primitive element theorem, splitting fields, algebraic closure, field embeddings and automorphisms solvability by radicals, Hilbert's theorem 90, norms and traces
2. Galois extensions, Galois groups, fundamental theorem of Galois theory
3. Finite fields

IV. Representations of Finite Groups:

1. Representations, characters, group algebras, orthogonality relations
2. Induced representations, Frobenius reciprocity, Burnside's theorem, representations of symmetric groups

V. Applications:

1. Codes
2. Cryptography, public-key cryptography, discrete logarithms, elliptic curves and cryptography
3. Polynomial algorithms and fast Fourier transforms.

[References]

1. Artin, M.: Algebra, Prentice Hall, 1991.
2. Hungerford, T. W.: Algebra, Springer, 1980.
3. Lang, S. : Algebra, 3rd ed., Springer, 2002.
4. Hardy, D.W., Richman, F. & Walker, C. L.: Applied Algebra: Codes, Ciphers and Discrete Algorithms, 2nd ed., Chapman & Hall, 2009.

Combinatorics Qualifying Exam Syllabus

1. Graphs

   Terminology of graphs and digraphs, Eulerian circuits, Hamiltonian circuits

2. Trees

   Cayley's theorem, spanning trees and greedy algorithms, search trees, strong connectivity

3. Colorings of graphs and Ramsey's theorem

   Brooks' theorem, Ramsey's theorem and Ramsey numbers, the Lóvasz sieve, the Erdős-Szekeres theorem

4. Systems of distinct representatives

   Bipartite graphs, Hall's condition, SDRs, König's theorem, Birkhoff's theorem

5. Dilworth's theorem and extremal set theory

   Partially ordered sets, Dilworth's theorem, Sperner's theorem, symmetric chains, the Erdös-Ko-Rado theorem

6. Flows in networks

   The Ford-Fulkerson theorem, the integrality theorem, a generalization of Birkhoff's theorem, circulations

7. The principle of inclusion and exclusion; inversion formulae

   Inclusion-exclusion, derangements, Euler and Möbius function, Möbius inversion, Burnside's lemma, problème des ménages

8. Permanents

   Bounds on permanents, Schrijver's proof of the Minc conjecture, Fekete's lemma, permanents of doubly stochastic matrices

9. The Van der Waerden conjecture

   The early results of Marcus and Newmann, London's theorem, Egoritsjev's proof

10. Elementary counting; Stirling numbers

    Stirling numbers of the first and second kind, Bell numbers, generating functions

11. Recursions and generating functions

    Elementary recurrences, Catalan numbers, counting of trees, Joyal theory, Lagrange inversion

12. Partitions

    The function $p_k(n)$, the partition function, Ferrers diagrams, Euler's identity, asymptotics, the Jacobi triple product identity, Young tableaux and the hook formula

13. Latin squares

    Orthogonal arrays, conjugates and isomorphism, partial and incomplete Latin squares, counting Latin squares, the Evans conjecture, the Dinitz conjecture

14. Hadamard matrices; Reed-Muller codes

    Hadamard matrices and conference matrices, recursive constructions, Paley matrices, Williamson's method, excess of a Hadamard matrix, first order Reed-Muller codes

15. Designs

    The Erdős-De Bruijn theorem, Steiner systems, balanced incomplete block designs, Hadamard designs, counting, (higher) incidence matrices, the Wilson-Petrenjuk theorem, symmetric designs, projective planes, derived and residual designs, the Bruck-Ryser-Chowla theorem, constructions of Steiner triple systems

16. Gaussian numbers and q-analogues

    Chains in the lattice of subspaces, q-analogue of Sperner's theorem, interpretation of the coefficients of the Gaussian polynomials, spreads

17. Lattices and Möbius inversion

    The incidence algebra of a poset, the Möbius function, chromatic polynomial of a graph, Weisner's theorem, complementing permutations of geometric lattices, connected labeled graphs, MDS codes

18. Pólya theory of counting

    The cycle index of a permutation group, counting orbits, weights, necklaces, the symmetric group, Stirling numbers

[Reference]

1. J. H. van Lint and R. M. Wilson. *A Course in Combinatorics*, second edition, Cambridge University Press, 2001.

資格考試參考資料－高等機率論
Advanced Probability Qualifying Exam Syllabus

1. Measure and Probability Space
2. Random Variables and distributions
3. Expected value
4. Random variables
5. Independence
6. Laws of large numbers
7. Convergence of random series
8. Weak convergence (convergence in distribution)
9. Characteristic functions
10. Central limit theorems
11. Markov chains and Random walks
12. Stationary measures
13. Recurrence and transience
14. Conditional expectation
15. Martingales, almost sure convergence
16. Doob's inequality, Conditional in L^p
17. Uniform integrability
18. Convergence in L^1
19. Definition and construction Brownian Motion

[References]

1. Richard Durrett, Probability: Theory and Examples, 4rd ed
2. Kai Lai Chung, A Course in Probability Theory, 2nd edition
3. Achim Klenke, Probability Theory: A comprehensive course
4. John B. Walsh, Knowing the Odds: An Introduction to Probability