

國立政治大學
110 學年度第一學期
微積分甲會考

作答注意事項

- 請先核對座位標示單及答案卷之姓名與學號正確無誤，如有錯誤，應於考試開始鈴響前舉手請監試人員處理。
- 測驗時間從下午 2:10 到 4:00，共計 110 分鐘。試卷加答案卷共 5 頁。
- 試卷包括選擇題與填充題，共有 20 個問題，總計 100 分，占學期成績之 30%。會考成績將作為微積分獎金給獎依據。
- 答題時請依題號作答，否則不予計分。
- This exam has 5 pages including the answer sheet. Look over your exam for discrepancies such as a missing page and make sure you have a complete exam package.
- Exam time is 2:10 pm—4:00 pm (Duration: 110 minutes).
- This exam includes multiple-choice questions and fill-in-the-blank questions. The total points of this exam are 100. This exam is worth 30% of your final grade and a scholarship will be awarded to students with excellent performance in this exam.
- Write each answer in the required space provided in the answer sheet. No point is earned when the answer is written in the wrong space.

考試科目 Course	微積分甲	開課 系級	應用數學系	日期 時間	111 年 1 月 8 日 14:10 至 16:00	試題編號 No.	A
----------------	------	----------	-------	----------	--------------------------------	-------------	---

注意事項

- 試題包括選擇題與填充題，共有 20 個問題，總計 100 分。
- 請在答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。
- 務必作答於答案卷，請勿作答於試題卷上，否則不予計分。

單選題 (multiplechoice questions) (共 10 題，每題 5 分，合計 50 分，答錯不倒扣)

1. (5 %) Which of the following integrals is the area of the region enclosed by $y = x$ and $y^2 = x + 6$?

A. $\int_{-2}^3 (y^2 - y - 6) dy.$

B. $\int_{-2}^3 (y - y^2 - 6) dy.$

C. $\int_{-6}^{-2} 2\sqrt{x+6} dx + \int_{-2}^3 (x - \sqrt{x+6}) dx.$

D. $\int_{-6}^{-2} 2\sqrt{x+6} dx + \int_{-2}^3 (\sqrt{x+6} - x) dx.$

2. (5 %) Suppose that $\lim_{x \rightarrow 0^+} f(x) = \alpha$ and $\lim_{x \rightarrow 0^-} f(x) = \beta$ where α and β are distinct real numbers.

Then

$$\lim_{x \rightarrow 0^-} f(-x \sin^2(1/x))$$

equals

- A. α . B. $-\alpha$. C. β . D. $-\beta$.

3. (5 %) Let

$$f(x) = \cos(2x) - \sin(2x) + 2 \cos x + 2 \sin x + 2x + 1, \quad \text{for } x \in [0, \pi].$$

Which of the following statements is TRUE?

A. f is increasing on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

B. f has a local minimum at $x = \frac{\pi}{4}$.

C. f has a local maximum at $x = \frac{2\pi}{3}$.

D. $\left(\frac{\pi}{2}, \pi + 2\right)$ is an inflection point of f .

4. (5 %) The limit $\lim_{x \rightarrow \infty} \left(\cos\left(\frac{1}{x}\right)\right)^{x^2}$ equals

- A. e . B. $e^{1/2}$. C. $e^{-1/2}$. D. 1.

命題老師：
(Teacher)

(簽章)

年

月

日

試題隨卷繳交

考試科目 Course	微積分甲	開課 系級	應用數學系	日期 時間	111 年 1 月 8 日 14:10 至 16:00	試題編號 No.	A
----------------	------	----------	-------	----------	--------------------------------	-------------	---

5. (5 %) The integral

$$\int_0^{\pi/3} \frac{\sec \theta \tan \theta}{1 + \sec \theta} d\theta$$

equals

A. 0. B. 1. C. $2 \ln 3$. D. $\ln 3 - \ln 2$.

6. (5 %) Assume that $f(0) = f(1) = 0$, f'' is continuous, and $\int_0^1 f''(x)f(x) dx = 1$. Then, the value

of $\int_0^1 f'(x)^2 dx$ is

A. 1. B. -1. C. 0. D. Cannot be determined.

7. (5 %) Which of the following statements is TRUE?

- A. If $f(x)$ is continuous on $(-\infty, \infty)$, then $f(x)$ has a derivative on $(-\infty, \infty)$.
- B. If $f(x)$ is continuous on $(-\infty, \infty)$, then $f(x)$ has an antiderivative on $(-\infty, \infty)$.
- C. If $f(-1) = -1$ and $f(1) = 1$, then $f(c) = 0$ for some c in $(-1, 1)$.
- D. If $f(x)$ is not continuous at 1, then $f(x)^2$ is not continuous at 1.

8. (5 %) If $y = \frac{\sqrt{x+2}(x+3)^5}{(x+4)^6}$, then y' equals

- A. $y' = y \left(\frac{1}{2(x+2)} + \frac{5}{x+3} - \frac{6}{x+4} \right)$.
- B. $y' = y \left(\frac{1}{2\sqrt{x+2}} + 5(x+3)^4 - 6(x+4)^5 \right)$.
- C. $y' = \frac{1}{2(x+2)} + \frac{5}{x+3} - \frac{6}{x+4}$.
- D. $y' = \frac{1}{2\sqrt{x+2}} + 5(x+3)^4 - 6(x+4)^5$.

9. (5 %) The integral

$$\int_0^1 \frac{1}{(1+x^2)^{3/2}} dx$$

equals

A. $1/2$. B. $\sqrt{2}/2$. C. $\sqrt{3}/2$. D. 1.

10. (5 %) Let f be a continuous function defined on $[1, \infty)$. Assume that that $\int_1^x f(t) dt = \sqrt{x}$. Then,

the value of $\int_1^3 f(x)^2 dx$ is

A. 3. B. $\frac{3}{4}$. C. $\frac{1}{4} \ln 3$. D. Cannot be determined.

命題老師：
(Teacher)

(簽章)

年

月

日

試題隨卷繳交

考試科目 Course	微積分甲	開課 系級	應用數學系	日期 時間	111 年 1 月 8 日 14:10 至 16:00	試題編號 No.	A
----------------	------	----------	-------	----------	--------------------------------	-------------	---

填充題 (fill-in-the-blank questions) (共 10 題, 每題 5 分, 合計 50 分, 答錯不倒扣)

11. (5 %) Evaluate the integral $\int_0^2 x\sqrt{1+2x^2} dx$.

12. (5 %) Find the area of the region bounded by the parabola $y = -x^2$, the tangent line to this parabola at $(1, -1)$, and the x -axis.

13. (5 %) The total production P of a certain product depends on the amount L of labor used and the amount K of capital investment. The production can be modeled by $P = L^{3/4}K^{1/4}$. If L and K satisfy the equation $3L + K = 10$ due to the total budget, find the maximum production P .

14. (5 %) Evaluate the limit

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k^4}{n^5} + \frac{k}{n^2} \right).$$

(Hint: You may recognize the sum as a Riemann sum.)

15. (5 %) Evaluate the integral

$$\int (\arcsin x)^2 dx.$$

16. (5 %) Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 6$ at $(1, 1)$.

17. (5 %) Find the triple (a, b, c) such that

$$\lim_{x \rightarrow 1} \frac{\sqrt{ax^2 + bx + c} - 3}{(x - 1)^2} = 5.$$

18. (5 %) Find the volume of the solid obtained by rotating the region bounded by the following curves about the specified axis:

$$x = (y - 3)^2, x = 4; \text{ about } y = 1.$$

19. (5 %) Use a linear approximation (or differentials) to give an estimation of the number $\sqrt{100.4}$.

20. (5 %) Let $f(x)$ be a continuous function on $(-\infty, \infty)$ such that

$$-x^2 + 5x - 1 \leq f(x) \leq x^2 - x + 3, \quad \text{for all } x < 1$$

and

$$-3x^3 + 2x^2 - 2x + 6 \leq f(x) \leq -4x^3 + 5x^2 + 3x - 1, \quad \text{for all } x > 1.$$

Find $f(1)$.

命題老師：
(Teacher)

(簽章)

年

月

日

試題隨卷繳交