國立政治大學 **109** 學年度第一學期 微積分甲會考 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 **Course**	微積分甲	開課系級	應用數學系	日期時間	**110** 年 **1** 月 **9** 日 **14:10** 至 **16:00**	試題編號 **No.**	

注意事項

- 試題包括選擇題與填充題，共有 **20** 個問題**,** 總計 **100** 分。
- 請在答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。
- 務必作答於答案卷，請勿作答於試題卷上，否則不予計分。

單選題 **(multiplechoice questions)** （共 **10** 題，每題 **5** 分，合計 **50** 分，答錯不倒扣**)**

1. (5 %) Let f be any continuous function defined on $(-\infty, \infty)$. Which of the following statements is True about f?

   A. f has a antiderivative on $(-\infty, \infty)$

   B. f has an derivative on $(-\infty, \infty)$

   C. $\dfrac{d}{dx}\displaystyle\int_0^1 f(x)dx = f(x)$

   D. If $\lim\limits_{x\to 0} f(x) = f(0)$, then $\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x}$ exists

2. (5 %) Consider the function

$$f(x) = \begin{cases} -x+1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

   Which of the following statements is True?

   A. $f(x)$ is discontinuous at $x = 1$ and the discontinuity at 1 is not removable.

   B. $f(x)$ is discontinuous at $x = 1$ but the discontinuity at 1 removable.

   C. $f(x)$ is continuous at $x = 1$ and $\lim\limits_{x\to 1} f(x)$ exists

   D. $f(x)$ is continuous at $x = 1$ but $\lim\limits_{x\to 1} f(x)$ does not exist

3. (5 %) Find an equation of the tangent line to the curve

$$y = \frac{x}{x-2}$$

   that are perpendicular (垂直) to the line $2x - y = 3$.

   A. $\dfrac{1}{2}x + y = 4$

   B. $\dfrac{1}{2}x + y = 2$

   C. $2x + y = 6$

   D. $2x + y = 9$

命題老師： **(Teacher)**	**(**簽章**)** 年 月 日	試題隨卷繳交

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4. (5 %) Find

$$\frac{d}{d\theta}\cos(\sec(\tan\theta))$$

A. $-\sin(\sec(\tan\theta))\sec(\tan\theta)\tan(\tan\theta)$

B. $-\sin(\sec(\tan\theta))\sec(\sec^2\theta)\tan(\sec^2\theta)$

C. $-\sin(\sec(\tan\theta))\sec(\tan\theta)\tan(\tan\theta)\sec^2\theta$

D. $-\sin(\sec(\tan\theta))\sec(\tan(\tan\theta)\sec^2\theta$

5. (5 %) Let $f(x)=\dfrac{\sqrt{x}}{x^2+3}$ on $[0,\infty)$. Which of the following statement is true?

A. f attains its absolute maximum value and absolute minimum value on $[0,\infty)$.

B. f does not have an absolute maximum or an absolute minimum at 0 because f is not differentiable at 0.

C. f does not have an absolute maximum because its domain is unbounded.

D. f is decreasing on $(0,\infty)$.

6. (5 %) If $f'(x)=\dfrac{-3(x+3)^5(x-2)^2}{(x+1)^3}$ for all x, then f has

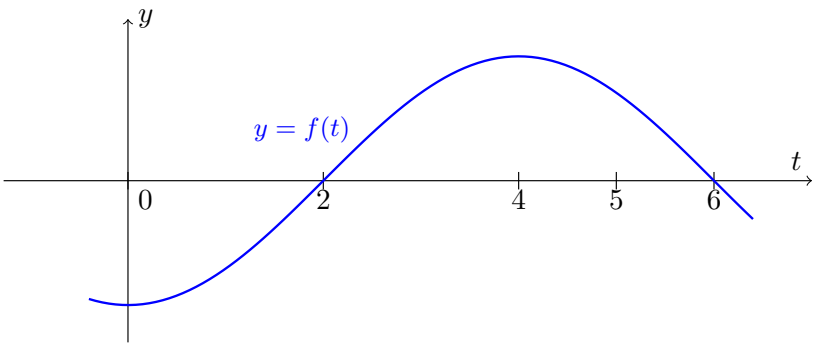
A. a local maximum at $x=2$.

B. a local minimum at $x=-1$.

C. a local maximum at $x=-3$.

D. a local minimum at $x=-3$.

7. (5 %) If $F(x)=\displaystyle\int_x^5 f(t)dt$, where f is the function whose graph is given, which of the following values is largest?



A. $F(0)$ B. $F(2)$ C. $F(4)$ D. $F(6)$

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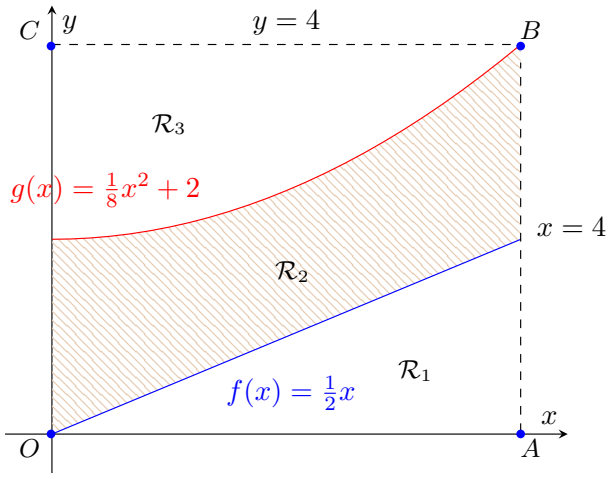
8. (5 %) The integral $\int \sin x \cos(\frac{1}{2}x)dx$ is

A. $-\frac{4}{3}\sin^3(\frac{1}{2}x)+C$ B. $-\frac{4}{3}\cos^3(\frac{1}{2}x)+C$

C. $-\frac{2}{3}\sin^3(\frac{1}{2}x)+C$ D. $-\frac{2}{3}\cos^3(\frac{1}{2}x)+C$

9. (5 %) Let $f(x)=\frac{1}{2}x$ and $g(x)=\frac{1}{8}x^2+2$ be functions defined in $[0,4]$. Consider regions $\mathcal{R}_1,\mathcal{R}_2,\mathcal{R}_3$ divided by $f(x)$ and $g(x)$ as follows.



Let $\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3$ be areas corresponding to $\mathcal{R}_1,\mathcal{R}_2,\mathcal{R}_3$, respectively. Which of the following statements is WRONG?

A. $\mathcal{A}_2=\int_0^4(\frac{1}{8}x^2+2-\frac{1}{2}x)\,dx$

B. $\mathcal{A}_2\geq\mathcal{A}_1$

C. The volume of the solid obtained by rotating about $\overline{OA}$ the region $\mathcal{R}_2$ is $V_2=\int_0^4\pi\left((\frac{1}{8}x^2+2)^2-(\frac{1}{2}x)^2\right)dx$.

D. The volume of the solid obtained by rotating about $\overline{BC}$ the region $\mathcal{R}_3$ is $V_3=\int_0^4 2\pi x(2-\frac{1}{8}x^2)dx$

10. (5 %) The integral $\int_0^2\frac{1}{(4+x^2)^{3/2}}dx$ is

A. $\frac{1}{8\sqrt{2}}$ B. $\frac{1}{4\sqrt{2}}$ C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{\sqrt{2}}$

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填充題 (fill-in-the-blank questions)（共 10 題，每題 5 分，合計 50 分，答錯不倒扣）

11. (5 %) Evaluate the limit: $\lim_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}$.

12. (5 %) If $\lim_{x\to 5+}\dfrac{f(x)+2}{x-5}=e^{\pi}$, then find the limit $\lim_{x\to 5+} f(x)$.

13. (5 %) Find $\dfrac{dy}{dx}$ if $y^2=x^2+\sin(xy)$.

14. (5 %) Find the derivative of the function

$$y=(\ln x)^{\cos x}$$

15. (5 %)

$$\begin{array}{ccc} f(1)=-1 & f'(1)=-2 & f''(1)=1 \\ g(1)=-2 & g'(1)=-4 & g''(1)=5 \\ h(1)=1 & h'(1)=3 & h''(1)=4 \end{array}$$

Given three twice-differentiable functions $f(x), g(x)$, and $h(x)$. The table above gives values for the functions and their first and second derivatives at $x=1$. Find

$$\lim_{x\to 1}\frac{(f(x))^3+h(x^2)}{xg(x)+2h(x)}.$$

16. (5 %) If a cubic (三次) polynomial $f(x)$ has a local minimum value $f(-5)=-5$ and (-2,4) is a point of inflection on the curve $y=f(x)$. Find $f(-1)$.

17. (5 %) Find the limit

$$\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{\frac{2}{n}}+\sqrt{\frac{4}{n}}+\ldots+\sqrt{\frac{2n}{n}}\right).$$

18. (5 %) Find the volume of the solid obtained by rotating about $y=3$ the region bounded by $x=y^2$ and $x=\dfrac{1}{2}y^2+2$.

19. (5 %) Evaluate the integral

$$\int_{\pi}^{0} e^{\cos x}\sin(2x)dx.$$

20. (5 %) Evaluate the integral

$$\int_1^2\frac{1-x-4x^2-x^3}{x^2(x+1)}dx.$$

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