國立政治大學　108 學年度第一學期　微積分甲會考 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 Course	微積分甲	開課 單位	應用數學系	日期 時間	**109 年 1 月 4 日 14:10 至 16:00**	試題編號 No.	

注意事項

- 試題包括選擇題與填充題，共有 **20** 個問題**,** 總計 **100** 分。
- 請在答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。
- 務必作答於答案卷，請勿作答於試題卷上，否則不予計分。

單選題 (multiple-choice questions)（共 **10** 題，每題 **5** 分，合計 **50** 分，答錯不倒扣）

1. (5 %) Which of the following statements is WRONG ?

   (A) A function f is called even if $f(-x) = f(x)$ for all x in its domain.

   (B) A function g is called odd if $g(-x) = -g(x)$ for all x in its domain.

   (C) The derivative of an even function is an odd function.

   (D) An antiderivative of an odd function is an even function.

2. (5 %) Let $f(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$, evaluate $(f^{-1})'(1)$.

   (A) 1 (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{4}$

3. (5 %) Let $f(x) = x^{2019} + x + 1$, which of the following statements is TRUE ?

   (A) $f(x)$ has a local maximum.

   (B) $f(x)$ has a local minimum.

   (C) $f(c) \geq 0$ for all $c \in \mathbb{R}$.

   (D) $f(x)$ does not have any local extrema on $\mathbb{R}$.

4. (5 %) Let $f(x) = \tan x$, which of the following statements is WRONG?

   (A) $\displaystyle\int (\tan x)^2 dx = \tan x - x + c$, where c is a constant.

   (B) There is a number c on $(0, \pi)$ such that $f'(c) = 0$.

   (C) f is continuous on $(0, \dfrac{\pi}{3})$.

   (D) $f(0) = f(\pi)$.

命題老師： (Teacher)	(簽章)	年　　月　　日	試題隨卷繳交

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5. (5 %) Find the area of the largest rectangle that can be inscribed in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

   (A) $\dfrac{ab}{4}$ (B) $\dfrac{ab}{2}$ (C) $2ab$ (D) $4ab$

6. (5 %) Evaluate $\lim\limits_{x \to 3} \left(\dfrac{x}{x-3} \int_3^x \dfrac{\sin(t)}{t} \right)$.

   (A) $-\sin(3)$

   (B) 0

   (C) $\sin(3)$

   (D) The limit does not exist.

7. (5 %) Two curves are orthogonal if their tangent lines are perpendicular at each point of intersection. Choose the two curves which are NOT orthogonal.

   (A) $x^2 + y^2 - 1 = 0$ and $x + y = 0$.

   (B) $x^2 + y^2 - x = 0$ and $x^2 + y^2 - y = 0$

   (C) $x^3 - y = 0$ and $x^2 + 6y - 7 = 0$

   (D) $x + 2y^2 - 10 = 0$ and $x + 5y - 10 = 0$

8. (5 %) Find the volume of the solid obtained by rotating about $x = 1$ the region between $y = x^2$ and $y = 3 - 2x^2$.

   (A) $\displaystyle\int_{-1}^{1} \pi(3 - 3x^2)^2 dx$

   (B) $\displaystyle\int_{0}^{1} 2\pi(1-x)(3 - 3x^2)dx + \int_{-1}^{0} 2\pi(2+\pi)(3 - 3x^2)dx$

   (C) $\displaystyle\int_{-1}^{1} 2\pi(1-x)(3 - 3x^2)dx$

   (D) $\displaystyle\int_{-1}^{1} 2\pi x(3 - 3x^2)dx$

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9. (5 %) Which of the following statements is WRONG?

(A) $\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$

(B) $\int f(x)g(x)dx = \left(\int f(x)dx \right)\left(\int g(x)dx \right)$

(C) $\int u\,dv = uv - \int v\,du$

(D) $\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b g(x)f'(x)dx$

10. (5 %) Which of the following statements is WRONG?

(A) $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$ for $n \neq -1$

(B) $\int \sec^2 x\,dx = \tan x + c$, where c is a constant.

(C) $\int \csc x\,dx = \ln|\csc x - \cot x| + c$ where c is a constant.

(D) $\int b^x dx = b^{x+1} + c$ for real number b and constant c.

填充題 (fill-in-the-blank questions) （共 10 題，每題 5 分，合計 50 分，答錯不倒扣）

11. (5 %) If the symbol $[\cdot]$ denotes the greatest integer function and $f(x) = [x] + [-x]$, find $\lim_{x \to 2} f(x)$ if it exists.

12. (5 %) Find the limit $\lim_{x \to \infty} (1 + \dfrac{1}{x} + \dfrac{2}{x^2} + ... + \dfrac{2019}{x^{2019}})^x$

13. (5 %) Find the linearization $L(x)$ of the function $f(x) = 2^x$ at $x = 1$.

14. (5 %) If $f(x) = e^x g(x)$, where $g(0) = 2$ and $g'(0) = 4$, find $f'(0)$.

15. (5 %) Find $\dfrac{dy}{dx}$ if $1 + xy = \cos(xy^2)$
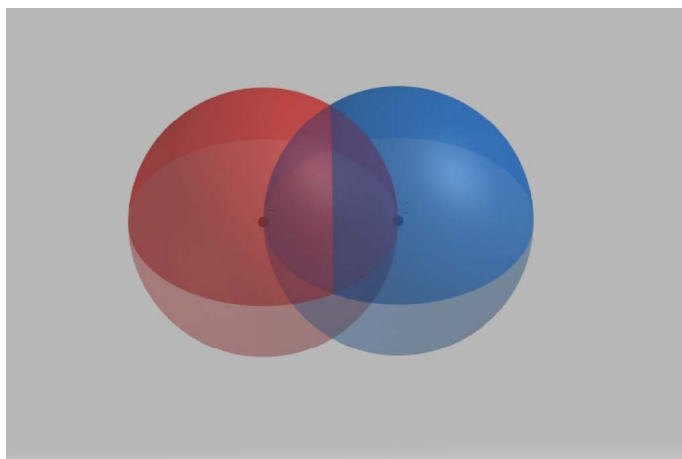
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16. (5 %) Find the critical numbers of $f(x) = x^4(x-1)^3$.

17. (5 %) Find $f(x)$, where $f''(x) = e^x + 2\sin x$, $f(0) = 3, f(\frac{\pi}{2}) = 0$.

18. (5 %) Find the area enclosed by the line $y = x - 1$ and the parabola $\frac{1}{2}y^2 = x + 3$.

19. (5 %) Find the volume common to two unit spheres if the center of each sphere lies on the surface of the other sphere.

20. (5 %) Find $\int \frac{108}{x^4 - 81} dx$.

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